6th Grade Math

Pacing Guide and Unpacked Standards

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Groveport Math Pacing Guide – Grade 6

➢ Indicates Blueprint Focus Standards

6th	Ratios & Proportional Relationships	The Number System	Expressions & Equations	Geometry	Statistics & Probability	Standards for Mathematical Practice
1st 9 Weeks	➢**6.RP.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities ➢**6.RP.2** Understand the concept of a unit rate a/b associated with a ratio ➢**6.RP.3 (a,b,c,d)** Use ratio and rate reasoning to solve real world and mathematical problems as it relates to measurement, pricing, constant speed, and rate	➢**6.NS.1** Interpret and compute quotients of fractions ➢**6.NS.2** Fluently divide multi-digit numbers using the standard algorithm ➢**6.NS.3** Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation ➢**6.NS.4** Find the greatest common factor of two whole numbers				**MP.1** Make sense of problems and persevere in solving them **MP.2** Reason abstractly and quantitatively **MP.3** Construct viable arguments and critique the

2nd 9 Weeks	➢**6.RP.3 (a,b,c,d)** Use ratio and rate reasoning to solve real world and mathematical problems as it relates to measurement, pricing, constant speed, and rate	➢**6.NS.5** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values ➢**6.NS.6(a,b,c)** Understand a rational number as a point on the number line. ➢**6.NS.7(a,c,d)** Understand ordering and absolute value of rational numbers ➢**6.NS.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane				reasoning of others **MP.4** Model with mathematics **MP.5** Use appropriate tools strategically **MP.6** Attend to precision **MP.7** Look for and make use of structure **MP.8** Look for and express regularity in repeated reasoning

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6th	Ratios & Proportional Relationships	The Number System	Expressions & Equations	Geometry	Statistics & Probability	Standards for Mathematical Practice
3rd 9 Weeks			➢**6.EE.1** Write and evaluate numerical expressions involving whole-number exponents ➢**6.EE.2(a,b,c)** Write, read, and evaluate expressions in which letters stand for numbers ➢**6.EE.3** Apply the properties of operations to generate equivalent expressions ➢**6.EE.4** Identify when two expressions are equivalent ➢**6.EE.5** Understand solving an equation or inequality as a process of answering a question ➢**6.EE.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem ➢**6.EE.7** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers ➢**6.EE.8** Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real world or mathematical problem ➢**6.EE.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another			**MP.1** Make sense of problems and persevere in solving them **MP.2** Reason abstractly and quantitatively **MP.3** Construct viable arguments and critique the reasoning of others **MP.4** Model with mathematics **MP.5** Use appropriate tools strategically **MP.6** Attend to precision

4th 9 Weeks				➢**6.G.1** Through composition into rectangles or decomposition into triangles, find the area of right triangles, other triangles, special quadrilaterals, and polygons; apply these techniques in the context of solving real-world and mathematical problems ➢**6.G.2** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism ➢**6.G.3** Draw polygons in the coordinate plane given coordinates for the vertices ➢**6.G.4** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures	➢**6.SP.1** Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers ➢**6.SP.2** Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape ➢**6.SP.3** Recognize the difference between measure of center measure of variation ➢**6.SP.4** Display numerical data in plots on a number line, including dot plots, histograms, and box plots ➢**6.SP.5** Summarize numerical data sets in relation to their context	**MP.7** Look for and make use of structure **MP.8** Look for and express regularity in repeated reasoning

Ohio's Learning Standards- Clear Learning Targets

Math, Grade 6

6.RP.1-3	Essential Understanding	Vocabulary
6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes." **6.RP.2** Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, "The recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar." We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." **6.RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagram, double number line diagrams, or equations.	Relations between two quantities can often be expressed as ratios and can be explained using ratio language Multiplication and division can be used to solve ratio and rate problems Ratios and rates apply to real life situations Percent is a rate of the number of units per 100 Multiplication and division can be used to generate equivalent ratios and rates Double number line diagrams and tape diagrams can show ratio relationships and be used to reason and solve real-world problems Graphical representation of an equivalent ratio will be linear	• Equivalent ratio • Rate • Ratio • Ratio table • Unit rate • Tape diagram • Double number line • part-to-part • part-to- whole • percent

Essential Skills

- I can write a ratio as a part-to-part or part-to-whole in different ways (a:b, a to b, a/b).
- I can explain what a part-to-part or part-to-whole relationship means in a given situation.
- I can explain that ratios compare two quantities and that the quantities do not have to be the same unit of measure.
- I can recognize that ratios appear in a variety of different contexts; part-to-whole, part-to-part, and rates.
- I can identify and calculate unit rate
- I can analyze the relationship between a ratio a:b and a unit rate a/b where b ≠ 0.
- I can make a table of equivalent ratios using whole numbers.
- I can find the missing values in a table of equivalent ratios.
- I can plot pairs of values that represent equivalent ratios on the coordinate plane.
- I can apply the concept of unit rate to solve real-world problems involving unit pricing.
- I can apply the concept of unit rate to solve real-world problems involving constant speed
- I can find a percent of a number as a rate per 100.
- I can solve real-world problems involving finding the whole, given a part and a percent.
- I can apply ratio reasoning to convert measurement units in real-world and mathematical problems.

Instructional Methods

Proportional reasoning is a process that requires instruction and practice. It does not develop over time on its own. Grade 6 is the first of several years in which students develop this multiplicative thinking. Examples with ratio and proportion must involve measurements, prices and geometric contexts, as well as rates of miles per hour or portions per person within contexts that are relevant to sixth graders. Experience with proportional and nonproportional relationships, comparing and predicting ratios, and relating unit rates to previously learned unit fractions will facilitate the development of proportional reasoning. Although algorithms provide efficient means for finding solutions, the cross-product algorithm commonly used for solving proportions will not aid in the development of proportional reasoning. Delaying the introduction of rules and algorithms will encourage thinking about multiplicative situations instead of indiscriminately applying rules.

Students develop the understanding that ratio is a comparison of two numbers or quantities. Ratios that are written as part-to-whole are comparing a specific part to the whole. Fractions and percents are examples of part-to-whole ratios. Fractions are written as the part being identified compared to the whole amount. A percent is the part identified compared to the whole (100). Provide students with multiple examples of ratios, fractions and percents of this type. For example, the number of girls in the class (12) to the number of students in the class (28) is the ratio 12 to 28.

Percents are often taught in relationship to learning fractions and decimals. This cluster indicates that percents are to be taught as a special type of rate. Provide students with opportunities to find percents in the same ways they would solve rates and proportions.

Part-to-part ratios are used to compare two parts. For example, the number of girls in the class (12) compared to the number of boys in the class (16) is the ratio the ratio 12 to 16. This form of ratios is often used to compare the event that can happen to the event that cannot happen. Rates, a relationship between two units of measure, can be written as ratios, such as miles per hour, ounces per gallon and students per bus. For example, 3 cans of pudding cost $2.48 at Store A and 6 cans of the same pudding costs $4.50 at Store B. Which store has the better buy on these cans of pudding? Various strategies could be used to solve this problem:

- A student can determine the unit cost of 1 can of pudding at each store and compare.
- A student can determine the cost of 6 cans of pudding at Store A by doubling $2.48.
- A student can determine the cost of 3 cans of pudding at Store B by taking ½ of $4.50.

Using ratio tables develops the concept of proportion. By comparing equivalent ratios, the concept of proportional thinking is developed and many problems can be easily solved.

Store A	
3 cans	6 cans
$2.48	$4.96

Store B	
6 cans	3 cans
$4.50	$2.50

Students should also solve real-life problems involving measurement units that need to be converted. Representing these measurement conversions with models such as ratio tables, t-charts or double number line diagrams will help students internalize the size relationships between same system measurements and relate the process of converting to the solution of a ratio

Multiplicative reasoning is used when finding the missing element in a proportion. For example, use 2 cups of syrup to 5 cups of water to make fruit punch. If 6 cups of syrup are used to make punch, how many cups of water are needed?

$$\frac{2}{5} = \frac{6}{x}$$

- Recognize that the relationship between 2 and 6 is 3 times; 2 · 3 = 6
- To find x, the relationship between 5 and x must also be 3 times. 3 · 5 = x, therefore, x = 15

$$\frac{2}{5} = \frac{6}{15}$$

The final proportion.

Other ways to illustrate ratios that will help students see the relationships follow.

Begin written representation of ratios with the words “out of” or “to” before using the symbolic notation of the colon and then the fraction bar; for example, 3 out of 7, 3 to 5, 6:7 and then 4/5.
Use skip counting as a technique to determine if ratios are equal.

Labeling units helps students organize the quantities when writing proportions.

$$\frac{3\ eggs}{2\ cups\ of\ flour} = \frac{z\ eggs}{8\ cups\ of\ flour}$$

Using hue/color intensity is a visual way to examine ratios of part-to-part. Students can compare the intensity of the color green and relate that to the ratio of colors used. For example, have students mix green paint into white paint in the following ratios: 1 part green to 5 parts white, 2 parts green to 3 parts white, and 3 parts green to 7 parts white. Compare the green color intensity with their ratios.

A rate is a ratio where two measurements are related to each other. When discussing measurement of different units, the word rate is used rather than ratio. Understanding rate, however, is complicated and there is no universally accepted definition. When using the term rate, contextual understanding is critical. Students need many opportunities to use models to demonstrate the relationships between quantities before they are expected to work with rates numerically.

A unit rate expresses a ratio as part-to-one or one unit of another quantity. Students understand the unit rate from various contextual situations. For example, if there are 2 cookies for 3 students, each student receives 2 /3 of a cookie, so the unit rate is 2:1. If a car travels 240 miles in 4 hours, the car travels 60 miles per hour (60:1).

Students will often use unit rates to solve missing value problems. Cost per item or distance per time unit are common unit rates, however, students should be able to flexibly use unit rates to name the amount of either quantity in terms of the other quantity. Students will begin to notice that related unit rates are reciprocals as in the first example. It is not intended that this be taught as an algorithm or rule because at this level, students should primarily use reasoning to find these unit rates.

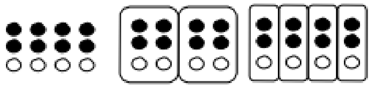
In Grade 6, students are not expected to work with unit rates expressed as complex fractions. Both the numerator and denominator of the original ratio will be whole numbers.

Examples:

A ratio is a comparison of two quantities or measures which can be written as a to b, a/b, or a:b. The comparison can be part-to- whole (ratio of guppies to all fish in an aquarium) or part-to-part (ratio of guppies to goldfish).

Students need to understand each of these ratios when expressed in the following forms: 6/15 , 6 to 15, or 6:15. These values can be simplified to, 2/5, 2 to 5, or 2:5; however, students would need to understand how the simplified values relate to the original numbers.

A comparison of 8 black circles to 4 white circles can be written as the ratio of 8:4 and can be regrouped into 4 black circles to 2 white circles (4:2) and 2 black circles to 1 white circle (2:1).



Students should be able to identify all these ratios and describe them using "For every…, there are…"

Example 1

A restaurant worker used 5 loaves of wheat bread and 2 loaves of rye bread to make sandwiches for an event.

- Write a ratio that compares the number of loaves of rye bread to the number of loaves of wheat bread.
- Describe what the ratio 7:2 means in terms of the loaves of bread used for the event.

Sample Solutions

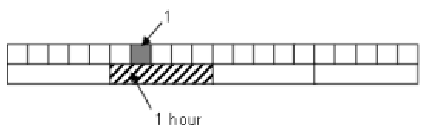
2:5

7:2 is the ratio of the total number of loaves of bread to the number of loaves of rye bread.

Example 2

On a bicycle you can travel 20 miles in 4 hours. What are the unit rates in this situation,?

Sample Solution: You can travel 5 miles in 1 hour written as $\frac{5mi}{1hr}$ and it takes $\frac{1}{5}$ of an hour to travel each mile. Students can represent the relationship between 20 miles and 4 hours.



Example 3

Using the information in the table, find the number of yards in 24 feet.

Feet	3	6	9	15	24
Yards	1	2	3	5	?

Solution

There are several strategies that students could use to determine the solution to this problem.

- Add quantities from the table to total 24 feet (9 feet and 15 feet); therefore the number of yards must be 8 yards (3 yards and 5 yards).
- Use multiplication to find 24 feet: 1) 3 feet x 8 = 24 feet; therefore 1 yard x 8 = 8 yards, or 2) 6 feet x 4 = 24 feet; therefore 2 yards x 4 = 8 yards.

Example 4

Compare the number of black to white circles. If the ratio remains the same, how many black circles will you have if you have 60 white circles?

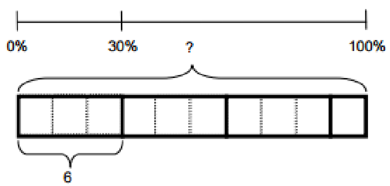
Black	4	40	20	60	?
White	3	30	15	45	60

Solution: 80 dots

Example 5

If 6 is 30% of a value, what is that value?

Solution: 20



Common Misconceptions / Challenges

Fractions and ratios may represent different comparisons. Fractions always express a part-to whole comparison, but ratios can express a part-to-whole comparison or a part-to-part comparison which can be written as: a to b, a/b, or a:b.

Even though ratios and fractions express a part-to-whole comparison, the addition of ratios and the addition of fractions are distinctly different procedures. When adding ratios, the parts are added, the wholes are added and then the total part is compared to the total whole. For example, (2 out of 3 parts) + (4 out of 5 parts) is equal to six parts out of 8 total parts (6 out of 8) if the parts are equal. When dealing with fractions, the procedure for addition is based on a common denominator: (2/3) + (4/5) = (10/15) + (12/15) which is equal to (22/15). Therefore, the addition process for ratios and for fractions is distinctly different.

Often there is a misunderstanding that a percent is always a natural number less than or equal to 100. Provide examples of percent amounts that are greater than 100%, and percent amounts that are less 1%.

Students often forget that a unit rate has a denominator of 1.

When completing ratio tables, remind students that they may to have to simplify a ratio with division before multiplying to find the unknown unit.

Students may invert ratios when trying to find an equivalent rate. Remind them to label the units in the numerator and denominator and then to keep the unit placement the same in the second ratio.

Criteria for Success (Performance Level Descriptors)

6.RP.1
- Limited: Write a ratio to describe a familiar relationship between two quantities using given information
- Basic: Write a ratio to describe a familiar relationship between two quantities
- Proficient: Write a ratio to describe a relationship between two quantities
- Accelerated: Select appropriate representations and strategies to solve mathematical and real-world ratio and rate problems
- Advanced: Select efficient representations and strategies to solve mathematical and real-world ratio and rate problems

6.RP.2
- Limited: N/A
- Basic: Solve familiar straightforward unit rate problems
- Proficient: Understand the concept of unit rates
- Accelerated:
- Advanced: N/A

6.RP.3
- Limited: Use models to solve simple problems involving ratios
- Basic: Solve routine straightforward problems involving ratios
- Proficient: Solve a wide variety of routine problems involving ratios and rates
- Accelerated: Solve a wide variety of problems involving ratios and rates
- Advanced: Solve a wide variety of real-world problems involving ratios and rates, including where a ratio is associated with a rate

6.RP.3a
- Limited: Complete simple ratio tables
- Basic: Find missing values in tables of equivalent ratios; Complete a table of familiar measurement unit conversions within the same system
- Proficient: Use ratio tables to solve routine real-world problems
- Accelerated: N/A
- Advanced: N/A

6.RP.3b
- Limited: Solve simple routine unit rate problems
- Basic: Solve routine straightforward real-world unit rate problems (including unit pricing)
- Proficient: Solve routine mathematical and real-world unit rate problems (including unit pricing and constant speed)
- Accelerated: N/A
- Advanced: N/A

6.RP.3c
- Limited: Find the percent of a quantity using 100 grids
- Basic: Find a percent of a quantity as a rate per 100 using 100 grids
- Proficient: Find a percent of a quantity as a rate per 100
- Accelerated: N/A
- Advanced: N/A

6.RP.3d
- Limited: N/A
- Basic: N/A
- Proficient: Convert measurement units within the same system using ratio reasoning
- Accelerated: Apply ratio reasoning to convert measurement units within the same system
- Advanced: Solve a variety of non-routine problems requiring conversion of measurement units

Prior Knowledge	Future Learning
4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison 5.NF.3 Understanding fractions as division	7.RP.1 Compute unit rates associated with ratios of fractions 7.RP.2 Recognize and represent proportional relationships between quantities 7.RP.3 Use proportional relationships to solve multistep ratio and percent problems

Career Connections

There are many careers that involve being able to set up, reduce, and solve for equivalent proportions, including:

- Actuaries
- Programmers
- Construction managers
- Insurance sales agents
- Dental assistants

Ohio's Learning Standards- Clear Learning Targets

Math, Grade 6

6.NS.1

6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?

Essential Understanding

Fractions can be divided visually and/or by using an algorithm

A fraction can be divided by another fraction by multiplying by the inverse

Vocabulary

- quotient
- fraction
- numerator
- denominator
- reduce/simplify
- reciprocal
- visual fraction model
- multiplicative inverse

Essential Skills

- I can compute quotients of fractions divided by fractions (including mixed numbers).
- I can interpret quotients of fractions.
- I can solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.
- I can create story contexts for problems involving the division of a fraction by a fractions

Instructional Methods

In 5th grade students divided whole numbers by unit fractions. Students continue this understanding by using visual models and equations to divide whole numbers by fractions and fractions by fractions to solve word problems. Students understand that a division problem such as 3 ÷ 2/5 is asking,“how many 2/5 are in 3?” One possible visual model would begin with three whole and divide each into fifths. There are 7 groups of two-fifths in the three wholes. However, one-fifth remains. Since one-fifth is half of a two-fifths group, there is a remainder of ½. Therefore, 3 ÷ 2/5 = 7 ½, meaning there are 7 ½ groups of two-fifths. Students interpret the solution, explaining how division by fifths can result in an answer with halves.

Students should also write contextual problems for fraction division problems. For example, the problem, 2/3 ÷ 1/6 can be illustrated with the following word problem: Susan has 2/3 of an hour left to make cards. It takes her about 1/6 of an hour to make each card. About how many can she make?
This problem can be modeled using a number line.

1. Start with a number line divided into thirds.

2. The problem wants to know how many sixths are in two-thirds. Divide each third in half to create sixths.

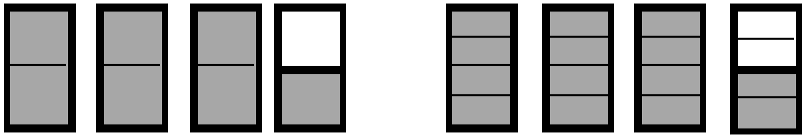
3. Each circled part represents 1/6. There are four sixths in two-thirds; therefore, Susan can make 4 cards.

Contexts and visual models can help students to understand quotients of fractions and begin to develop the relationship between multiplication and division. Model development can be facilitated by building from familiar scenarios with whole or friendly number dividends or divisors. Computing quotients of fractions build upon and extends student understandings developed in Grade 5. Students make drawings, model situations with manipulatives, or manipulate computer generated models.

Computation with fractions is best understood when it builds upon the familiar understandings of whole numbers and is paired with visual representations. Solve a simpler problem with whole numbers, and then use the same steps to solve a fraction divided by a fraction. Looking at the problem through the lens of “How many groups?” or “How many in each group?” helps visualize what is being sought.

For example: 12÷3 means; How many groups of three would make 12? Or how many in each of 3 groups would make 12? Thus 7/2 ÷ 1/4 can be solved the same way. How many groups of ¼ make 7/2? Or, how many objects in a group when 7/2 fills one fourth?

Creating the picture that represents this problem makes seeing and proving the solutions easier.



Set the problem in context and represent the problem with a concrete or pictorial model. 5/4÷1/2
5/4 cups of nuts fills 1/2 of a container. How many cups of nuts will fill the entire container?

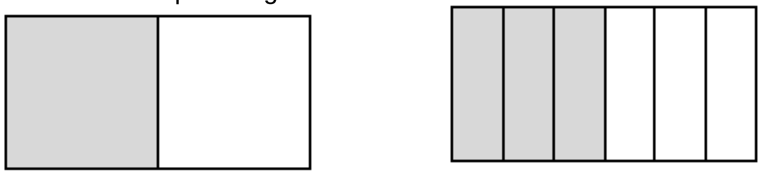
Teaching “invert and multiply” without developing an understanding of why it works first, leads to confusion as to when to apply the shortcut.

Learning how to compute fraction division problems is one part, being able to relate the problems to real-world situations is important. Providing opportunities to create stories for fraction problems or writing equations for situations is essential.

Example 1

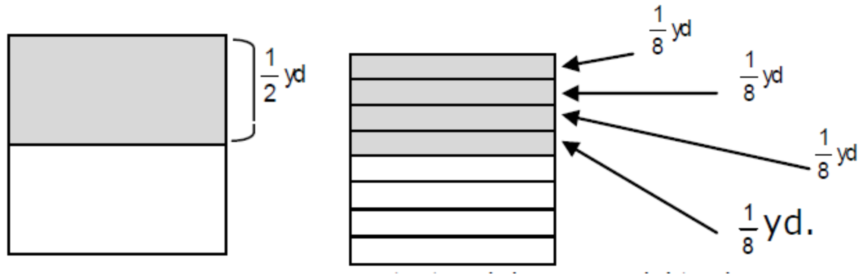
3 people share ½ pound of chocolate. How much of a pound of chocolate does each person get?

Solution: Each person gets 1/6 lb. of chocolate.



Example 2
Manny has ½ yard of fabric to make book covers. Each book is made from 1/8 yard of fabric. How many book covers can Manny make?

Solution: Manny can make 4 book covers.



Example 3
Represent ½ ÷ 2/3 in a problem context and draw a model to show your solution.

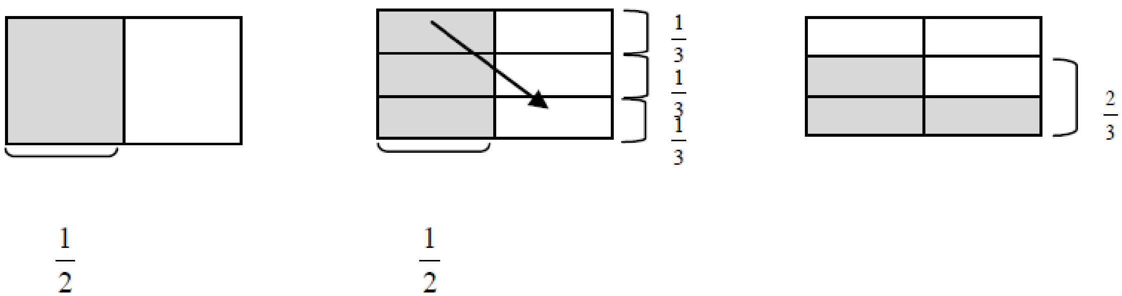
Solution: You are making a recipe that calls for 2/3 cup of yogurt. You have ½ cup of yogurt from a snack pack. How much of the recipe can you make?

Explanation of model:

The first model shows ½ cup. The shaded squares in all three models show ½ cup.
The second model shows ½ cup and also shows 1/3 cups horizontally.
The third model shows ½ cup moved to fit in only the area shown by 2/3 of the model.

2/3 is the new referent unit (whole).

3 out of the 4 squares in the 2/3 portion are shaded. A ½ cup is only ¾ of a 2/3cup portion, so you can only make ¾ of the recipe.



Common Misconceptions/Challenges

- Students may believe that dividing by 1/2 is the same as dividing in half. Dividing by half means to find how many 1/2s there are in a quantity, whereas, dividing in half means to take a quantity and split it into two equal parts. Thus 7 divided by 1/2 = 14 and 7 divided in half equals 3 1/2.
- Students may confuse the order for the dividend and the divisor.
- Students may use the inverse of the dividend rather than the divisor.

Criteria for Success (Performance Level Descriptors)

- **Limited:** Recognize a visual model for division of a fraction by a fraction
- **Basic:** Interpret a visual model for division of a fraction by a fraction
- **Proficient:** Divide a fraction by a fraction using a visual model
- **Accelerated:** Divide a fraction by a fraction using visual models and equations
- **Advanced:** Divide a fraction by a fraction

Prior Knowledge	Future Learning
5. NF.B.7- Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.	7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

Ohio's Learning Standards- Clear Learning Targets

Math, Grade 6

6.NS.2-4

6.NS.2 Fluently divide multi-digit numbers using a standard algorithm.

6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation.

6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*

Essential Understanding

Knowledge and understanding of place value is necessary for operations with decimals

The same algorithms used to multiple and divide whole numbers can be applied to operations with decimals

The greatest common factor of two whole numbers is the largest number that divides both numbers evenly

The least common multiple is the smallest positive number that is a multiple of two or more numbers.

Vocabulary

- quotient
- product
- sum
- Difference
- numerator
- denominator
- divisor
- dividend
- factor
- multiple
- greatest common factor
- lowest common multiple

Essential Skills

- I can divide multi-digit numbers using a standard algorithm with speed and accuracy, without any math tools (i.e., calculator, multiplication chart).
- I can fluently add multi-digit decimals using a standard algorithm.
- I can subtract multi-digit decimals using a standard algorithm
- I can divide multi-digit decimals using a standard algorithm
- I can multiply multi-digit decimals using a standard algorithm
- I can identify the factors of two whole numbers less than or equal to 100 and determine the Greatest Common Multiple.
- I can identify the multiples of two whole numbers less than or equal to 12 and determine the Least Common Multiple.
- I can apply the Distributive Property to rewrite addition problems by factoring out the Greatest Common Factor.

Instructional Methods

6.NS.2

Students are expected to fluently and accurately divide multi-digit whole numbers. Divisors can be any number of digits at this grade level. As students divide they should continue to use their understanding of place value to describe what they are doing. When using the standard algorithm, students' language should reference place value. For example, when dividing 32 into 8456, as they write a 2 in the quotient they should say, "there are 200 thirty-twos in 8456" and could write 6400 beneath the 8456 rather than only writing 64.

6.NS.3

This standard calls for students to fluently compute with decimals. A companion of fluency is the extension of the students' existing number sense to decimals. It is insufficient to merely teach procedures about "where to move the decimal." Rather, the focus of instruction and student work should be on operations and number sense. The use of estimation strategies supports student understanding of operating on decimals. Example: First, students estimate the sum and then find the exact sum of 14.4 and 8.75. An estimate of the sum might be 14 + 9 or 23. Students may also state if their estimate is low or high. They would expect their answer to be greater than 23. They can use their estimates to self-correct. Answers of 10.19 or 101.9 indicate that students are not considering the concept of place value when adding (adding tenths to tenths or hundredths to hundredths) whereas answers like 22.125 or 22.79 indicate that students are having difficulty understanding how the four-tenths and seventy- five hundredths fit together to make one whole and 25 hundredths. Students use the understanding they developed in 5th grade related to the patterns involved when multiplying and dividing by powers of ten to develop fluency with operations with multi-digit decimals.

Explanations and Examples:

The next two tasks in are not examples of tasks asking students to compute using the standard algorithms for multiplication and division because most people know what those kinds of problems look like. Instead, these tasks show what kinds of reasoning and estimation strategies students need to develop in order to support their algorithmic computations.

- Use the fact that 13 × 17 = 221 to find the following.
  a) 13 × 1.7
  b) 130 × 17
  c) 13 × 1700
  d) 1.3 × 1.7
  e) 2210 ÷ 13
  f) 22100 ÷ 17
  g) 221 ÷ 1.3

All these solutions use the associative and commutative properties of multiplication (explicitly or implicitly).

a) 13 × 1.7 = 13 × (17 × 0.1) = (13 × 17) × 0.1, so the product is one-tenth the product of 13 and 17. In other words, 13 × 1.7 = 22.1
b) Since one of the factors is ten times one of the factors in 13×17, the product will be ten times as large as well: 130 × 17 = 2210
c) 13 × 1700 = 13 × (17 × 100) = (13 × 17) × 100, so 13 × 1700 = 22100
d) Since each of the factors is one tenth the corresponding factor in 13 × 17, the product will be one one hundredth as large: 1.3 × 1.7 = 2.21
e) 2210 ÷ 13 = ? is equivalent to 1 3× ? = 2210. Since the product is ten times as big and one of the factors is the same, the other factor must be ten times as big. So 2210 ÷ 13 = 170
f) As in the previous problem, the product is 100 times as big, and since one factor is the same, the other factor must be 100 times as big: 22100÷17=1300
g) 221 ÷ 1. 3= ? is equivalent to 1.3 × ? = 221. Since the product is the same size and one of the factors is one-

- Place a decimal on the right side of the equal sign to make the equation true. Explain your reasoning for each.

1. 3.58 × 1.25 = 044750
2. 26.97 ÷ 6.2 = 04350

Solution: Reasoning from the meanings of division and multiplication

1. 3.58 × 1.25 =4 .475. We are multiplying a number between 3 and 4 by a number a little more than. More specifically, we can appeal to the meaning of multiplication and ask, “How many 3.58’s do we have?” A little more than one of them. Thus, the product must be a number around 4. We can also say that the product must be greater than 3 × 1 = 3 and less than 4 × 2 = 8. Assuming the digits shown are correct, the only place one could put the decimal that would result in a value between 3 and 8 would be 4.475.
2. 26.97 ÷ 6.2 = 4.35. We are dividing a number around 27 by a number a little more than 6. More specifically, we can appeal to the meaning of division and ask, “How many 6.2’s go into 26.97?” Since 4 sixes go into 24, and 5 sixes go into 30. Thus, it is reasonable for the quotient to be a number around 4.5.

6.NS.4

Greatest common factor and least common multiple are usually taught as a means of combining fractions with unlike denominators. This cluster builds upon the previous learning of the multiplicative structure of whole numbers, as well as prime and composite numbers in Grade 4. Although the process is the same, the point is to become aware of the relationships between numbers and their multiples. For example, consider answering the question: “If two numbers are multiples of four, will the sum of the two numbers also be a multiple of four?” Being able to see and write the relationships between numbers will be beneficial as further algebraic understandings are developed. Another focus is to be able to see how the GCF is useful in expressing the numbers using the distributive property, (36 + 24) = 12(3+2), where 12 is the GCF of 36 and 24. Students often confuse the concepts of factors and multiples. One effective way to avoid this confusion is to consistently use the vocabulary of factors and multiples each and every time students work on multiplication and division (i.e. the numbers being multiplied are the factors; the product is the multiple).

Explanations and Examples:

Students will find the greatest common factor of two whole numbers less than or equal to 100. For example, the greatest common factor of 40 and 16 can be found by:

1) listing the factors of 40 (1, 2, 4, 5, 8, 10, 20, 40) and 16 (1, 2, 4, 8, 16), then taking the greatest common factor (8). Eight (8) is also the largest number such that the other factors are relatively prime (two numbers with no common factors other than one). For example, 8 would be multiplied by 5 to get 40; 8 would be multiplied by 2 to get 16. Since the 5 and 2 are relatively prime, then 8 is the greatest common factor. If students think 4 is the greatest, then show that 4 would be multiplied by 10 to get 40, while 16 would be 4 times 4. Since the 10 and 4 are not relatively prime (have 2 in common), the 4 cannot be the greatest common factor.

2) listing the prime factors of 40 (2 • 2 • 2 • 5) and 16 (2 • 2 • 2 • 2) and then multiplying the common factors (2 • 2 • 2 = 8).

Students also understand that the greatest common factor of two prime numbers will be 1.

Students use the greatest common factor and the distributive property to find the sum of two whole numbers. For example, 36 + 8 can be expressed as

4 (9 + 2) = 4 (11).

Students find the least common multiple of two whole numbers less than or equal to twelve. For example, the least common multiple of 6 and 8 can be found by:

1) listing the multiplies of 6 (6, 12, 18, 24, 30, …) and 8 (8, 26, 24, 32, 40…), then taking the least in common from the list (24); or
2) using the prime factorization.

Step 1: Find the prime factors of 6 and 8.

6 = 2 • 3
8 = 2 • 2 • 2

Step 2: Find the common factors between 6 and 8. In this example, the common factor is 2

Step 3: Multiply the common factors and any extra factors: 2 • 2 • 2 • 3 or 24 (one of the twos is in common; the other twos and the three are the extra factors.

- What is the greatest common factor (GCF) of 24 and 36? How can you use factor lists or the prime factorizations to find the GCF
  Solution: $2^2 \cdot 3 = 12$ Students should be able to explain that both 24 and 36 have 2 factors of 2 and one factor of 3, thus 2 x 2 x 3 is the greatest common factor

- What is the least common multiple (LCM) of 12 and 8? How can you use multiple lists or the prime factorizations to find the LCM?

Solution: $2^3 \cdot 3 = 24$. Students should be able to explain that the least common multiple is the smallest number that is a multiple of 12 and a multiple of 8. To be a multiple of 12, a number must have 2 factors of 2 and one factor of 3(2 x 2 x 3). To be a multiple of 8, a number must have 3 factors of 2 (2 x 2 x 2). Thus the least common multiple of 12 and 8 must have 3 factors of 2 and one factor of 3 (2 x 2 x 2 x 3).

- Rewrite 84 + 28 by using the distributive property. Have you divided by the largest common factor? How do you know?

- Given various pairs of addends using whole numbers from 1-100, students should be able to identify if the two numbers have a common factor. If they do, they identify the common factor and use the distributive property to rewrite the expression. They prove that they are correct by simplifying both expressions.

  27 + 36 = 9 (3 + 4)
  63 = 9 x 7
  63 = 63

  31 + 80

  There are no common factors. I know that because 31 is a prime number, it only has 2 factors, 1 and 31. I know that 31 is not a factor of 80

  because 2 x 31 is 62 and 3 x 31 is 93.

Common Misconceptions / Challenges

When adding and subtracting decimals, students may forget that place values must line up. For example, when adding 3.45 and 6, students may put the 6 under the 5 instead of writing 6 as 6.00.

When multiplying decimals, students may not place the decimal in the correct place in the product

When dividing decimals, students may not remember that the divisor may not have a decimal point when using the algorithm

Students confuse the terms factor and multiple

Students multiply the two numbers together when finding least common multiple

Criteria for Success (Performance Level Descriptors)

- Limited: Divide simple multi-digit whole numbers; Add, subtract, and multiply multi-digit whole numbers and decimals to hundredths using strategies and algorithms
- Basic: Divide multi-digit whole numbers; Add and subtract multi-digit decimal numbers; Divide multi-digit decimals by whole number divisors; Find common factors of two numbers less than or equal to 100; Find common multiples of two numbers less than or equal to 12
- Proficient: Add, subtract, multiply, and divide multi-digit decimals; Find the greatest common factor of two numbers less than or equal to 100; Find the least common multiple of two whole numbers less than or equal to 12
- Accelerated: Add, subtract, multiply, and divide multi-digit decimals to solve real-world problems; Use least common multiples and greatest common factors to solve routine real-world problems
- Advanced: Efficiently use least common multiples and greatest common factors to solve real-world problems

Prior Knowledge

5.NBT.B.6-Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors

5.NBT.B.7-Add, subtract, multiply, and divide decimals to hundredths

Future Learning

7.NS.A.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers

7.NS.A.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers

7.NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers

Career Connections

- Architects
- Chemical engineers
- Surveyors
- Accountants
- Researchers
- Doctors
- Chefs / cooks
- Nurses
- Stock brokers

Ohio's Learning Standards- Clear Learning Targets

Math, Grade 6

6.NS.5	Essential Understanding	Vocabulary
6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values, e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge; use positive and negative numbers to represent quantities in real world contexts, explaining the meaning of 0 in each situation	The system of rational numbers includes negative numbers as well as positive numbers Rational numbers can be represented in multiple ways and are useful when examining situations involving numbers that are not whole. Rational numbers can be used to indicate change with a fixed reference point	• Integer • Positive • Negative • quantities • opposite (as in direction) • sea level • elevation • absolute value • credit • debit

Essential Skills

- I can use integers to represent quantities in real world situations (above/below sea level, etc.).
- I can explain where zero fits into a situation represented by integers.
- I can describe and give examples of how positive or negative numbers are used to describe quantities having opposite directions or opposite values
- I can recognize that positive and negative signs represent opposite values and/or directions

Instructional Methods

Students should use rational numbers (fractions, decimals, and integers) to represent real-world contexts and understand the meaning of 0 in each situation. It should be explained to students that zero can have different meanings based on the context of the problem. Sometimes zero represents an amount that does not change, like the line of scrimmage in a football game. Other times, it represents the number zero, like with temperature. You should also discuss a variety of words that they may encounter and whether those words indicate a positive or negative situation. Some examples include deposit/withdraw, above par/below par, above sea level/below sea level, gain/loss, above freezing/below freezing, dropped/rose, winning/losing, forward/back, spending/earning.

Example 1:

a) Use an integer to represent 25 feet below sea level

b) Use an integer to represent 25 feet above sea level

c) What would 0 (zero) represent in the scenario above?

Solution:

a) -25

b) +25

c) 0 would represent sea level

Example 2

Jackson owes his sister Monica $15. Explain the meaning of zero in this situation

Solution:

The integer zero represents no longer owing his sister money.

Common Misconceptions / Challenges

Some students have difficulty understanding what zero represents in certain real-life situations (sometimes it represents an amount that does not change and other times it is used to represent real world ideas, such as sea level)

Some of the situations presented students aren't familiar with or have had no experience with, therefore they don't understand the meanings (ie. Golf, stock market, football line of scrimmage)

Criteria for Success (Performance Level Descriptors)

- Limited: Can identify or locate a positive or negative whole number on a number line
- Basic: Use number lines to compare and order positive and negative numbers
- Proficient: Use number lines to compare and order positive and negative numbers
- Accelerated: Use number lines to compare and order positive and negative numbers
- Advanced: N/A

Prior Knowledge	**Future Learning**
3.NF.A.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram	7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Career Connections

Most careers out there deal with numbers in some way, and many deal with positive and negative quantities. Some include:

- Statisticians
- Weather Forecasters
- Bankers
- Bookkeepers
- Map makers
- Physicists
- Tax preparers

Ohio's Learning Standards- Clear Learning Targets

Math, Grade 6

6.NS.6

6**.NS.6** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.
b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

Essential Understanding

Rational numbers can be arranged in order

Points can be graphed in all for quadrants of a coordinate grid by using ordered pairs to determine location

Recognize that if a<b, then –a>-b because a number and its opposite are equal distances from zero

Moving along a horizontal number line to the right means that numbers are increasing

Recognize that when two ordered pairs differ only by signs of the coordinates, then the locations of the points are related by reflections across one or both axes

Vocabulary

- rational number
- coordinate plane
- quadrant
- opposite
- ordered pair
- integer
- reflection

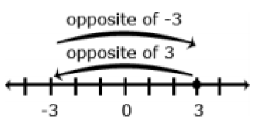
Essential Skills

- I can show and explain why every rational number can be represented by a point on a number line
- I can plot a number and its opposite on a number line and recognize that they are equidistant from zero
- I can find the opposite of any given number including zero
- I can use the signs of the coordinates to determine the location of an ordered pair in the coordinate plane
- I can reason about the location of two ordered pairs that have the same values but different signs
- I can plot a point on a number line or coordinate plane
- I can read a point from a number line or coordinate plane

Instructional Methods

In elementary school, students worked with positive fractions, decimals and whole numbers on the number line. In 6th grade, students extend the number line to represent all rational numbers and recognize that number lines may be either horizontal or vertical (i.e. thermometer). Students recognize that a number and its opposite are equidistance from zero (reflections about the zero). The opposite sign (–) shifts the number to the opposite side of 0. For example, – 4 could be read as “the opposite of 4” which would be negative 4. The following example, – (–6.4) would be read as “the opposite of the opposite of 6.4” which would be 6.4. Zero is its own opposite. Students worked with Quadrant I in elementary school. As the x-axis and y-axis are extending to include negatives, students begin with the Cartesian Coordinate system. Students recognize the point where the x-axis and y-axis intersect as the origin. Students identify the four quadrants and are able to identify the quadrant for an ordered pair based on the signs of the coordinates. For example, students recognize that in Quadrant II, the signs of all ordered pairs would be (–, +). Students understand the relationship between two ordered pairs differing only by signs as reflections across one or both axes. For example, in the ordered pairs (-2, 4) and (-2, -4), the y-coordinates differ only by signs, which represents a reflection across the x-axis. A change is the x-coordinates from (-2, 4) to (2, 4), represents a reflection across the y-axis. When the signs of both coordinates change, [(2, -4) changes to (-2, 4)], the ordered pair has been reflected across both axes. Students are able to plot all rational numbers on a number line (either vertical or horizontal) or identify the values of given points on a number line. For example, students are able to identify where the following numbers would be on a number line: –4.5, 2, 3.2, –3 3/5, 0.2, –2, 11/2.

Number lines can be used to show numbers and their opposites. Both 3 and -3 are 3 units from zero on the number line. Graphing points and reflecting across zero on a number line extends to graphing and reflecting points across axes on a coordinate grid. The use of both horizontal and vertical number line models facilitates the movement from number lines to coordinate grids.



Example 1

Graph the following points in the correct quadrant of the coordinate plane. If you reflected each point across the x-axis, what are the coordinates of the reflected points? What similarities do you notice between coordinates of the original point and the reflected point?

$\left(\frac{1}{2}, -3\frac{1}{2}\right)$ $\left(-\frac{1}{2}, -3\right)$ $(0.25, -0.75)$

Solution: The coordinates of the reflected points would be (1/2, 3 1/2) (-1/2, 3) (0.25, -.75) Note that the y-coordinates are opposite.

Example 2: What is the opposite of 2 ½? Explain your answer?

Solution: -2 ½ because it is the same distance from 0 on the opposite side.

Students place the following numbers would be on a number line: -4.5

Example 3

Place the following in order from least to greatest; -4.4, 2, 3.2, -3 3/5, 0.2, -2, 11/2.

Solution:

The numbers in order from least to greatest are: -4.4, -3 3/5, -2, 0.2, 2, 3.2, 11/2. Students place each of these numbers on a number line to justify this order.

Common Misconceptions / Challenges
Students often place −1 3/4 between −1 and 0 instead of between −2 and −1. Students have difficulty determining the opposites when it is written with parenthesis (ex. –(-(-8))) Students move along the y axis before moving along the x axis when plotting coordinates Students may number the quadrants wrong. Students may reflect over the wrong axis

Criteria for Success (Performance Level Descriptors)
Limited: N/A **Basic:** N/A **Proficient:** N/A **Accelerated:** N/A **Advanced:** N/A

Prior Knowledge	Future Learning
5.G.A.1- Understand how a coordinate system works 5.G.A.2- Graphing points in the first quadrant of the coordinate plane	7.NS.1.b. Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

Career Connections
- Statisticians - Weather Forecasters - Financial planners - Mathematicians - Actuaries / Insurance workers - Physicists - Political observers / pollsters - Doctors and nurses

Ohio's Learning Standards- Clear Learning Targets

Math, Grade 6

6.NS.7	Essential Understanding	Vocabulary
6.NS.7 **6.NS.7** Understanding ordering and absolute value of rational numbers. **a**: Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. **b**: Write, interpret, and explain statements of order for rational numbers in real-world contexts. **c**: Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. **d**: Distinguish comparisons of absolute value from statements about order.	Absolute value can be described in more than one way, depending on the context. It can be distance, or it can be size (magnitude) Inequalities can be written by placement of numbers on a number line. Numbers farther to the right are greater than numbers to the left.	• ordering • rational number • absolute value • magnitude

Essential Skills

- I can interpret statements of inequality as statements about relative position of two numbers on a number line diagram.
- I can write, interpret, and explain statements of order for rational numbers in real-world contexts
- I can identify absolute value of rational numbers.
- I can interpret absolute value as magnitude for a positive or negative quantity in a real-world situation
- I can distinguish comparisons of absolute value from statements about order and apply to real world contexts.

Instructional Methods

Explanations and Examples:

a. Students identify the absolute value of a number as the distance from zero but understand that although the value of -7 is less than -3, the absolute value (distance) of -7 is greater than the absolute value (distance) of -3. Students use inequalities to express the relationship between two rational numbers, understanding that the value of numbers is smaller moving to the left on a number line. For example, −4 1/2 < −2 because −4 1/2 is located to the left of –2 on the number line.

b. Students write statements using < or > to compare rational number in context. However, explanations should reference the context rather than “less than” or “greater than”. For example, the balance in Sue’s checkbook was –12.55. The balance in Ron’s checkbook was –10.45. Since –12.55 < –10.45, Sue owes more than Ron. The interpretation could also be “Ron owes less than Sue”.

c. Students understand absolute value as the distance from zero and recognize the symbols | | as representing absolute value. For example, | –7 | can be interpreted as the distance –7 is from 0 which would be 7. Likewise | 7 | can be interpreted as the distance 7 is from 0 which would also be 7. In real- world contexts, the absolute value can be used to describe size or magnitude. For example, for an ocean depth of –900 feet, write | –900| = 900 to describe the distance below sea level.

d. When working with positive numbers, the absolute value (distance from zero) of the number and the value of the number is the same; therefore, ordering is not problematic. However, negative numbers have a distinction that students need to understand. As the negative number increases (moves to the left on a number line), the value of the number decreases. For example, –24 is less than –14 because –24 is located to the left of –14 on the number line. However, absolute value is the distance from zero. In terms of absolute value (or distance) the absolute value of –24 is greater than –14. For negative numbers, as the absolute value increases, the value of the number decreases.

Common models to represent and compare integers include number line models, temperature models and the profit loss model. On a number line model, the number is represented by an arrow drawn from zero to the location of the number on the number line; the absolute value is the length of this arrow. The number line can also be viewed as a thermometer where each point of on the number line is a specific temperature. In the profit-loss model, a positive number corresponds to profit and the negative number corresponds to a loss. Each of these models is useful for examining values but can also be used in later grades when students begin to perform operations on integers. In working with number line models, students internalize the order of the numbers; larger numbers on the right or top of the number line and smaller numbers to the left or bottom of the number line. They use the order to correctly locate integers and other rational numbers on the number line. By placing two numbers on the same number line, they are able to write inequalities and make statements about the relationships between the numbers.

Case 1: Two positive numbers

5 > 3

5 is greater than 3

Case 2: One positive and one negative number

3 > -3

positive 3 is greater than negative 3

negative 3 is less than positive 3

Case 3: Two negative numbers

-3 > -5 negative 3 is greater than negative 5

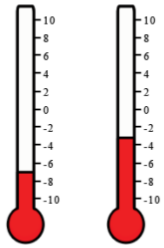
negative 5 is less than negative 3

Example 1
Write a statement to compare – 4 . and –2. Explain your answer.

Solution: – 4 . < –2 because – 4 . is located to the left of –2 on the number line
Students recognize the distance from zero as the absolute value or magnitude of a rational number. Students need multiple experiences to understand the relationships between numbers, absolute value, and statements about order.
Comparative statements generate informal experience with operations and lay the foundation for formal work with operations on integers in grade 7.

6.NS.7b Explanation
Students write statements using < or > to compare rational number in context. However, explanations should reference the context rather than "less than" or "greater than".
Example1:
The balance in Sue's checkbook was –$12.55. The balance in Ron's checkbook was –$10.45. Write an inequality to show the relationship between these amounts. Who owes more?

Solution: –12.55 < –10.45, Sue owes more than Ron. The interpretation could also be Ron owes less than Sue".
Example 2:
One of the thermometers shows -3°C and the other shows -7°C. Which thermometer shows which temperature? Which is the colder temperature? How much colder? Write an inequality to show the relationship between the temperatures and explain how the model shows this relationship.



Solution:
- The thermometer on the left is -7; right is -3
- The left thermometer is colder by 4 degrees
- Either -7 < -3 or -3 > -7

Example 3:
A meteorologist recorded temperatures in four cities around the world. List these cities in order from coldest temperature to warmest temperature:

	Solution:
Albany 5°	Juneau -9°
Anchorage -6°	Buffalo -7°
Buffalo -7°	Anchorage -6°
Juneau -9°	Albany- 5°
Reno 12°	Reno- 12°

6.NS.7c Explanation
Students recognize the distance from zero as the absolute value or magnitude of a rational number and recognize the symbols | | as representing

In real-world contexts, the absolute value can be used to describe size or magnitude. For example, for an ocean depth of 900 feet, write | –900| = 900 to describe the distance below sea level.

Example 1: Which numbers have an absolute value of 7?
Solution: 7 and –7 since both numbers have a distance of 7 units from 0 on the number line.

Example 2: What is the | –3 ½|?
Solution: 3 ½

6.NS.7d Explanation
When working with positive numbers, the absolute value (distance from zero) of the number and the value of the number is the same; therefore, ordering is not problematic. However, negative numbers have a distinction that students need to understand. As the negative number increases (moves to the left on a number line), the value of the number decreases. For example, –24 is less than –14 because –24 is located to the left of –14 on the number line. However, absolute value is the distance from zero. In terms of absolute value (or distance) the absolute value of –24 is greater than the absolute value of –14. For negative numbers, as the absolute value increases, the value of the negative number decreases.

Common Misconceptions / Challenges

Students often forget that the farther left a number is on the number line, then the smaller that number is

Students may not understand that distance cannot be negative, therefore absolute value is always positive or zero

Criteria for Success (Performance Level Descriptors)

6.NS.7a

- **Limited:** N/A
- **Basic:** N/A
- **Proficient:** N/A
- **Accelerated:** N/A
- **Advanced:** N/A

6.NS.7b

- **Limited:** N/A
- **Basic:** N/A
- **Proficient:** N/A
- **Accelerated:** N/A
- **Advanced:** Write, interpret, and explain statements of order in real-world contexts

6.NS.7c

- **Limited:** N/A
- **Basic:** Use positive and negative numbers to represent quantities in real-world contexts; Represent real-world quantities with positive and negative numbers
- **Proficient:** Represent real-world quantities with positive and negative numbers
- **Accelerated:** Represent real-world quantities with positive and negative numbers
- **Advanced:** Represent real-world quantities with positive and negative numbers

6.NS.7d

- **Limited:** N/A
- **Basic:** N/A
- **Proficient:** N/A
- **Accelerated:** N/A
- **Advanced:** N/A

Prior Knowledge	Future Learning
	7.NS.1.c Understand subtraction of rational numbers as adding the additive inverse, p − q = p + (−q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

Career Connections:

- Electricians
- Weather Forecasters
- Researchers
- Geologists
- Actuaries / Insurance workers
- Physicists
- Statisticians
- Doctors and nurses

Ohio's Learning Standards- Clear Learning Targets

Math, Grade 6

6.NS.8

<u>6.NS.8</u> Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Essential Understanding

The x (horizontal) and y (vertical) axes intersect at point (0, 0), forming four quadrants.

Use the coordinate plane to graph points, line segments and geometric shapes in the various quadrants and then use the absolute value to find the related distances.

Vocabulary

- coordinate plane
- axis / axes
- ordered pair
- quadrant
- absolute value
- solve
- find distance

Essential Skills

- I can calculate absolute value.
- I can graph points in all four quadrants of the coordinate plane.
- I can solve real-world problems by graphing points in all four quadrants of a coordinate plane.
- I can calculate the distances between two points with the same first coordinate or the same second coordinate using absolute value, given only coordinates.

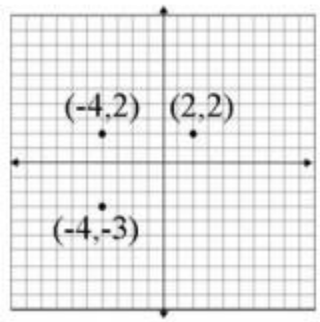
Instructional Methods
Students find the distance between points whose ordered pairs have the same x-coordinate (vertical) or same y-coordinate (horizontal). For example, the distance between (–5, 2) and (–9, 2) would be 4 units.

This would be a horizontal line since the y-coordinates are the same. In this scenario, both coordinates are in the same quadrant. The distance can be found by using a number line to find the distance between –5 and –9. Students could also recognize that –5 is 5 units from 0 (absolute value) and that –9 is 9 units from 0 (absolute value). Since both of these are in the same quadrant, the distance can be found by finding the difference between 9 and 5. (|9| − |5|).

Coordinates could also be in two quadrants. For example, the distance between (3, –5) and (3, 7) would be 12 units. This would be a vertical line since the x-coordinates are the same. The distance can be found by using a number line to count from –5 to 7 or by recognizing that the distance (absolute value) from –5 to 0 is 5 units and the distance (absolute value) from 0 to 7 is 7 units so the total distance would be 5 + 7 or 12 units.

Example 1

If the points on the coordinate plane below are the three vertices of a rectangle, what are the coordinates of the fourth vertex? How do you know? What are the length and width of the rectangle?



Solution:

To determine the distance along the x-axis between the point (-4, 2) and (2, 2) a student must recognize that -4 is |−4| or 4 units to the left of 0 and 2 is |2| or 2 units to the right of zero, so the two points are total of 6 units

Common Misconceptions / Challenges

Points with a x or y value of 0 are not in a quadrant.

Students may have difficulty finding the distance between two points when one is negative and the other is positive. They often times subtract the two numbers.

When both values are positive, students sometimes add the numbers together to find the distance rather than subtract. (ex. (3,5) and (3, 8) students will add 8+5 and tell that the distance is 13)

Criteria for Success (Performance Level Descriptors)

6.NS.8

- **Limited:** Draw polygons in the coordinate plane given coordinates in the first quadrant
- **Basic:** Find and position positive and negative rational numbers on a horizontal or vertical number line and on a coordinate plane; Locate points in all four quadrants of the coordinate plane; Solve routine real-world and mathematical problems by graphing points in the first quadrant
- **Proficient:** Locate points and ordered pairs in all four quadrants of the coordinate plane (understand the signs in ordered pairs); Solve routine real-world and mathematical problems by graphing points in the first quadrant
- **Accelerated:** Locate points and ordered pairs in all four quadrants of the coordinate plane (understand the signs in ordered pairs); Solve real world and mathematical problems by graphing points in the first quadrant
- **Advanced:** Graph ordered pairs in all four quadrants of the coordinate plane

Prior Knowledge

5.G.A.1- Understand how a coordinate system works

5.G.A.2- Graphing points in the first quadrant of the coordinate plane

Future Learning

7.NS.1.c Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

Career Connections

Careers using graphing include:

- Financial managers
- Weather forecasters
- Computer programmers
- Biological scientists
- Actuaries / Insurance workers
- Physicists
- Statisticians
- Economists

For a more complete list, check out: http://www.xpmath.com/careers/math_topics.php

Ohio's Learning Standards- Clear Learning Targets

Math, Grade 6

6.EE.1-4	Essential Understanding	Vocabulary
6.EE.1 Write and evaluate numerical expressions involving whole-number exponents. **6.EE.2** Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, using the algebraic order of operations when there are no parentheses to specify a particular order. For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$. **6.EE.3** Apply the properties of operations to generate equivalent expressions. **6.EE.4** Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).	Numbers and variables are not like terms. Students should understand that a letter represents one number in an expression. When that number replaces the letter, the expression can be evaluated to one number. Students understand that a base number can be represented with a positive whole number, positive fraction, or positive decimal and that for any number a, we define am to be the product of m factors of a. The number a is the base and m is called the exponent or power of a.	• base • exponent • evaluate • sum • difference • term • product • quotient • coefficient • algebraic expression • power • factor • numeric expression • equivalent • substitute • expression • Identity property • Associative property • Distributive property • Commutative property

Essential Skills

- I can write expressions involving whole number exponents.
- I can evaluate expressions involving whole number exponents.
- I can explain the meaning of a number raised to a power
- I can translate a relationship given in words into an algebraic expression
- I can identify parts of an expression by using correct mathematical terms
- I can recognize an expression as both a single value and as two or more terms on which an operation is performed
- I can evaluate an algebraic expression for a given value
- I can substitute values in formulas to solve real-world problems
- I can apply the order of operations when evaluating both arithmetic and algebraic expression
- I can apply the properties of operations to generate equivalent expressions
- I can determine whether two expressions are equivalent by using the same value to evaluate both expressions
- I can use the properties of operations to justify that two expressions are equivalent

Instructional Methods

The skills of reading, writing and evaluating expressions are essential for future work with expressions and equations, and are a Critical Area of Focus for Grade 6. In earlier grades, students added grouping symbols ( ) to reduce ambiguity when solving equations. Now the focus is on using ( ) to denote terms in an expression or equation. Students should now focus on what terms are to be solved first rather than invoking the PEMDAS rule. Likewise, the division symbol (3 ÷ 5) was used and should now be replaced with a fraction bar (3/5). Less confusion will occur as students write algebraic expressions and equations if x represents only variables and not multiplication. The use of a dot (•) or parentheses between numbers is preferred.

Provide opportunities for students to write expressions for numerical and real-world situations. Write multiple statements that represent a given algebraic expression. For example, the expression x – 10 could be written as “ten less than a number,” “a number minus ten,” “the temperature fell ten degrees,” “I scored ten fewer points than my brother,” etc. Students should also read an algebraic expression and write a statement.

Through modeling, encourage students to use proper mathematical vocabulary when discussing terms, factors, coefficients, etc.

Provide opportunities for students to write equivalent expressions, both numerically and with variables. For example, given the expression $x + x + x + x + 4 \cdot 2$, students could write 2x + 2x + 8 or some other equivalent expression. Make the connection to the simplest form of this expression as 4x + 8. Because this is a foundational year for building the bridge between the concrete concepts of arithmetic and the abstract thinking of algebra, using hands-on materials (such as algebra tiles, counters, unifix cubes, "Hands on Algebra") to help students translate between concrete numerical representations and abstract symbolic representations is critical.

Provide expressions and formulas to students, along with values for the variables so students can evaluate the expression. Evaluate expressions using the order of operations with and without parentheses.

Include whole- number exponents, fractions, decimals, etc. Provide a model that shows step-by-step thinking when simplifying an expression. This demonstrates how two lines of work maintain equivalent algebraic expressions and establishes the need to have a way to review and justify thinking.

Provide a variety of expressions and problem situations for students to practice and deepen their skills. Start with simple expressions to evaluate and move to more complex expressions. Likewise start with simple whole numbers and move to fractions and decimal numbers. The use of negatives and positives should mirror the level of introduction in Grade 6 The Number System; students are developing the concept and not generalizing operation rules.

6.EE.1

Students demonstrate the meaning of exponents to write and evaluate numerical expressions with whole number exponents. The base can be a whole number, positive decimal or a positive fraction (i.e. $\frac{1}{2}^4$ can be written ½ •½ • ½ • ½ which has the same value as 1). Students recognize that an expression with a variable represents the same mathematics (i.e. x^4 can be written as x • x • x • x) and write algebraic expressions from verbal expressions.

Example 1

Write the following as a numerical expressions using exponential notation.

- o The area of a square with a side length of 8 m Solution: $(8m)^2$
- o The volume of a cube with a side length of 5 ft.: Solution: 5^3 ft^3
- o Yu-Lee has a pair of mice. The mice each have 2 babies. The babies grow up and have two babies of their own:

Solution: 32 mice

Example 2

Evaluate: 4^3

Solution: 64

Evaluate: $5+2^4$ x 6

Solution: 101

6.EE.2

Students write expressions from verbal descriptions using letters and numbers. Students understand order is important in writing subtraction and division problems. Students understand that the expression “5 times any number, n” could be represented with 5n and that a number and letter written together means to multiply.

Students use appropriate mathematical language to write verbal expressions from algebraic expressions. Students can describe expressions such as 3 (2 + 6) as the product of two factors: 3 and (2 + 6). The quantity (2 + 6) is viewed as one factor consisting of two terms.

Students evaluate algebraic expressions, using order of operations as needed. Given an expression such as 3x + 2 y, find the value of the expression when x is equal to 4 and y is equal to 2.4. This problem requires students to understand that multiplication is understood when numbers and variables are written together and to use the order of operations to evaluate.

$$3 \cdot 4 + 2 \cdot 2.4$$

$$12 + 4.8$$

$$16.8$$

Given a context and the formula arising from the context, students could write an expression and then evaluate for any number. For example, it costs $100 to rent the skating rink plus $5 per person. The cost for any number (n) of people could be found by the expression, 100 + 5n. What is the cost for 25 people?

It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number.

- r + 21 as “some number plus 21 as well as “r plus 21”
- n • 6 as “some number times 6 as well as “n times 6”
- $\frac{s}{6}$ and s ÷ 6 as “as some number divided by 6” as well as “s divided by 6”

Students should identify the parts of an algebraic expression including variables, coefficients, constants, and the names of operations (sum, difference, product, and quotient). Development of this common language helps students to understand the structure of expressions and explain their process for simplifying expressions.

Terms are the parts of a sum. When the term is an explicit number, it is called a constant. When the term is a product of a number and a variable, the number is called the coefficient of the variable.

Variables are letters that represent numbers. There are various possibilities for the numbers they can represent; students can substitute these possible numbers for the letters in the expression for various different purposes.

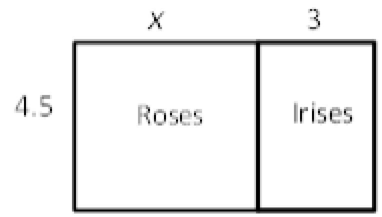
Consider the following expression: $X^2+5y+3x+6$

- The variables are x and y.
- There are 4 terms: x^2 , 5y, 3x, and 6.
- There are 3 variable terms, x^2 , 5y, 3x. They have coefficients of 1, 5, and 3 respectively. The coefficient of x2 is 1, since x2 = 1 x2 . The term 5y represent 5 y's or 5 * y.
- There is one constant term, 6.
- The expression shows a sum of all four terms.

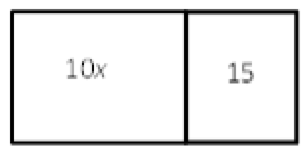
Examples:

- 7 more than 3 times a number (Solution: $3x + 7$)
- 3 times the sum of a number and 5 (Solution: $3(x + 5)$)
- 7 less than the product of 2 and a number (Solution: $2x – 7$)
- Twice the difference between a number and 5 (Solution: $2(z – 5)$)
- Evaluate $5(n + 3) – 7n$, when $n = ½$
- The expression $c + 0.07c$ can be used to find the total cost of an item with 7% sales tax, where c is the pre-tax cost of the item. Use the expression to find the total cost of an item that cost $25.
- The perimeter of a parallelogram is found using the formula $p = 2l + 2w$. What is the perimeter of a rectangular picture frame with dimensions of 8.5 inches by 11 inches.

6.EE.3
Students use the distributive property to write equivalent expressions. For example, area models from elementary can be used to illustrate the distributive property with variables. Given that the width is 4.5 units and the length can be represented by x + 3, the area of the flowers below can be expressed as 4.5(x + 3) or 4.5x + 13.5.



When given an expression representing area, students need to find the factors. For example, the expression 10x + 15 can represent the area of the figure below. Students find the greatest common factor (5) to represent the width and then use the distributive property to find the length (2x + 3). The factors (dimensions) of this figure would be 5(2x + 3).



6.EE.4

Students demonstrate an understanding of like terms as quantities being added or subtracted with the same variables and exponents. For example, 3x + 4x are like terms and can be combined as 7x; however, 3x + 4x2 are not. This concept can be illustrated by substituting in a value for x. For example, 9x – 3x = 6x not 6. Choosing a value for x, such as 2, can prove non-equivalence.

9(2) – 3(2) = 6(2) 18 – 6 = 12 12 = 12	however	9(2) – 3(2) = 6 18– 6 = 6 12 ≠ 6

Students connect their experiences with finding and identifying equivalent forms of whole numbers and can write expressions in various forms. Students generate equivalent expressions using the associative, commutative, and distributive properties. They can prove that the expressions are equivalent by simplifying each expression into the same form.

Example 1

Are the expressions equivalent? How do you know?

4m + 8   4(m+2)   3m + 8 + m   2 + 2m + m + 6 + m

Solution

Expression	Simplifying the Expression	Explanation
4m + 8	*4m + 8*	Already in simplest form
4(m+2)	*4(m+2)* *4m + 8*	*Distributive property*
3m + 8 + m	*3m + 8 + m* *3m + m + 8* *(3m + m) + 8* *4m + 8*	*Combined like terms*
2 + 2m + m + 6 + m	*2 + 2m + m + 6 + m* *2 + 6 + 2m + m + m* *(2 + 6) + (2m + m + m)* *8 + 4m* *4m + 8*	*Combined like terms*

Common Misconceptions / Challenges

Many of the misconceptions when dealing with expressions stem from the misunderstanding/reading of the expression. For example, knowing the operations that are being referenced with notation like, x^3, 4x, 3(x + 2y) is critical. The fact that x^3 means $x \cdot x \cdot$ x, not 3 x or 3 times x; 4 x means 4 times x or $x + x + x + x$, not forty-something.

One common mistake is in translating the phrase “5 less than a number”. Students often write “5 – x” instead of “x – 5”.

When evaluating 4x when x = 7, substitution does not result in the expression meaning 47.

When using the distributive property, students will often multiply the first term, but forget to do the same to the second term.

Students assume if there is not a coefficient in front of a variable, there is not actually a number there. They do not see that y = 1y.

When solving equations and inequalities, they may use the inverse operation on only one side and on the other or they may use the same operation rather than the inverse.

Criteria for Success (Performance Level Descriptors)

6.EE.1
- **Limited:** Evaluate numerical expressions with two operations
- **Basic:** Write and evaluate numerical expressions with up to two operations including those with exponents of 2 and 3
- **Proficient:** Write and evaluate numerical expressions including those with whole number exponents
- **Accelerated:** Write expressions and equations that correspond to given situations
- **Advanced:** Write expressions and equations for complex mathematical and real-world situations

6.EE.2
- **Limited:** Evaluate one-step expressions
- **Basic:** Evaluate algebraic expressions with up to two operations; Identify up to two-step equivalent expressions
- **Proficient:** Evaluate algebraic expressions
- **Accelerated:** Evaluate complex algebraic expressions including exponents
- **Advanced:** N/A

6.EE.2a
- **Limited:** Identify expressions and equations that correspond to given routine situations
- **Basic:**
- **Proficient:** Write expressions and equations that correspond to given routine situations
- **Accelerated:** N/A
- **Advanced:** N/A

6.EE.2b
- **Limited:** N/A
- **Basic:** Identify one- and two-step expressions and equations that correspond to given familiar situations
- **Proficient:** N/A
- **Accelerated:** N/A
- **Advanced:** N/A

6.EE.2c
- **Limited:** N/A
- **Basic:** N/A
- **Proficient:** N/A
- **Accelerated:** N/A
- **Advanced:** N/A

6.EE.3
- **Limited:** N/A
- **Basic:** N/A
- **Proficient:** N/A
- **Accelerated:** N/A
- **Advanced:** N/A

6.EE.4

- **Limited:** Apply the understanding of equivalent expressions to identify equivalent expressions; Use substitution to determine whether a given number makes an equation true
- **Basic:**
- **Proficient:** Apply the understanding of equivalent expressions to identify equivalent expressions
- **Accelerated:** Apply the properties of operations to write equivalent expressions
- **Advanced:** Explain why two expressions are equivalent using precise mathematical language

Prior Knowledge	**Future Learning**
5.NBT.2 Use whole-number exponents to denote powers of 10. 5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. 5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.	7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. 7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

Career Connections

There are many careers that involve being able to set up, reduce, and solve for expressions and equations, including:

- Construction worker
- Nurse
- Bridge builder
- Doctors
- Cashiers
- Landscapers

For a more information, click the link: http://www.homeschoolmath.net/teaching/why_need_square_roots.php

Ohio's Learning Standards- Clear Learning Targets

Math, Grade 6

6.EE.5

6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Essential Understanding

Students understand that an inequality with numerical expressions is either true or false. It is true if the numbers calculated on each side of the inequality sign result in a correct statement and false otherwise.

Students understand solving an inequality is answering the question of which values from a specified set, if any, make the inequality true.

Vocabulary

- equation
- inequality
- substitution
- set
- solution

Essential Skills

- I can explain that solving an equation or inequality lead to finding the value(s) of the variable that will make it a true mathematical statement
- I can use the solution to an equation or inequality to prove that the answer is correct.
- I can substitute a given value into an algebraic equation or inequality to determine whether it is part of the solution set

Instructional Methods

In order for students to understanding equations: The skill of solving an equation must be developed conceptually before it is developed procedurally. This means that students should be thinking about what numbers could possibly be a solution to the equation before solving the equation. For example, in the equation x + 21 = 32 students know that 21 + 9 = 30 therefore the solution must be 2 more than 9 or 11, so x = 11.

Provide multiple situations in which students must determine if a single value is required as a solution, or if the situation allows for multiple solutions. This creates the need for both types of equations (single solution for the situation) and inequalities (multiple solutions for the situation). Solutions to equations should not require using the rules for operations with negative numbers since the conceptual understanding of negatives and positives is being introduced in Grade 6. When working with inequalities, provide situations in which the solution is not limited to the set of positive whole numbers but includes rational numbers. This is a good way to practice fractional numbers and introduce negative numbers. Students need to be aware that numbers less than zero could be part of a solution set for a situation. As an extension to this concept, certain situations may require a solution between two numbers. For example, a problem situation may have a solution that requires more than 10 but not greater than 25. Therefore, the exploration with students as to what this would look like both on a number line and symbolically is a reasonable extension.

Students explore equations as expressions being set equal to a specific value. The solution is the value of the variable that will make the equation or inequality true. Students use various processes to identify the value(s) that when substituted for the variable will make the equation true.

Example 1:
Joey had 26 papers in his desk. His teacher gave him some more and now he has 100. How many papers did his teacher give him?

This situation can be represented by the equation 26 + n = 100 where n is the number of papers the teacher gives to Joey. This equation can be stated as "some number was added to 26 and the result was 100." Students ask themselves, "What number was added to 26 to get 100?" to help them determine the value of the variable that makes the equation true. Students could use several different strategies to find a solution to the problem:

- Reasoning: 26 + 70 is 96 and 96 + 4 is 100, so the number added to 26 to get 100 is 74.
- Use knowledge of fact families to write related equations: n + 26 = 100, 100 - n = 26, 100 - 26 = n. Select the equation that helps to find n easily.
- Use knowledge of inverse operations: Since subtraction "undoes" addition then subtract 26 from 100 to get the numerical value of n.
- Scale model: There are 26 blocks on the left side of the scale and 100 blocks on the right side of the scale. All the blocks are the same size. 74 blocks need to be added to the left side of the scale to make the scale balance.
- Bar Model: Each bar represents one of the values. Students use this visual representation to demonstrate that 26 and the unknown value together make 100.

<table>
<tr><td colspan="2">100</td></tr>
<tr><td>26</td><td>n</td></tr>
</table>

Solution

Students recognize the value of 74 would make a true statement if substituted for the variable.

26 + n = 100

26 + 74 = 100

100 = 100

Example 2

The equation 0.44 s = 11 where s represents the number of stamps in a booklet. The booklet of stamps costs 11 dollars and each stamp costs 44 cents. How many stamps are in the booklet? Explain the strategies used to determine the answer. Show that the solution is correct using substitution.

Solution

There are 25 stamps in the booklet. I got my answer by dividing 11 by 0.44 to determine how many groups of 0.44 were in 11.

By substituting 25 in for s and then multiplying, I get 11.

0.44(25) = 11

11 = 11

Example 3:

Twelve is less than 3 times another number can be shown by the inequality 12 < 3n. What numbers could possibly make this a true statement?

Solution:

Since 3 • 4 is equal to 12 I know the value must be greater than 4. Any value greater than 4 will make the inequality true. Possibilities are 4.13, 6, 5 3/4, and 200. Given a set of values, students identify the values that make the inequality true.

Example 4

Given the expression x + 2 ½ which of the following value(s) for x would make x + 2 ½ = 6?

{ 0,3 } ½ , 4

Solution

By using substitution, students identify 3 ½ as the value that will make both sides of the equation equal.

Example 5

Find the value(s) of x that will make x + 3.5 ≥ 9. {5, 5.5, 6, 15/2 , 10.2,15}

Solution

Using substitution, students identify 5.5, 6, 15 2 , 10.2, and 15 as the values that make the inequality true. NOTE: If the inequality had been x + 3.5 > 9, then 5.5 would not work since 9 is not greater than 9.

Common Misconceptions / Challenges

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Sometimes students forget balance an equation (performing the same operation on both sides) Many teachers draw a line down through the equal sign (or inequality) and stress that whatever gets written on one side must be written on the other side.

Students fail to see juxtaposition (side by side) as indicating multiplication. For example, evaluating 3x as 35 when x = 5 instead of 3 times 5 = 15. Also, students may rewrite 8 – 2a as 6a.

Criteria for Success (Performance Level Descriptors)

- **Limited:** Solve simple one-step equations involving addition and subtraction
- **Basic:** N/A
- **Proficient:** N/A
- **Accelerated:** N/A
- **Advanced:** N/A

Prior Knowledge	Future Learning
5.OA.A.1. Use parentheses in numerical expressions, and evaluate expressions with this symbol. Formal use of algebraic order of operations is not necessary. 5.OA.A.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.	7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities

Career Connections

Many jobs are out there that require the ability to solve equations and inequalities. Careers include:

- Accountants
- Counter and rental clerks
- Engineers (all types)
- Doctors / Nurses
- Order processors
- Electricians
- Mechanics
- Credit card workers
- Doctors
- Nurses
- Stock brokers

Ohio's Learning Standards- Clear Learning Targets
Math, Grade 6

6.EE.6 **<u>6.EE.6</u>** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	**<u>Essential Understanding</u>** Use variables to write expressions involving addition and subtraction, multiplication and division from real-world problems. Evaluate these expressions when given the value of the variable.	**<u>Vocabulary</u>** • unknown • variable • constant • algebraic expression • set

<u>Essential Skills</u>

- I can use a variable to write an algebraic expression that represents a real-world situation when a specific number is unknown
- I can explain and give examples of how a variable can represent a single unknown number or can represent any number in a specified set
- I can use a variable to write an expression that represents a consistent relationship in a particular pattern

Instructional Methods

Students write expressions to represent various real-world situations. For example, the expression a + 3 could represent Susan's age in three years, when a represents her present age. The expression 2n represents the number of wheels on any number of bicycles. Other contexts could include age (Johnny's age in 3 years if a represents his current age) and money (value of any number of quarters).

Given a contextual situation, students define variables and write an expression to represent the situation. For example, the skating rink charges $100 to reserve the place and then $5 per person. Write an expression to represent the cost for any number of people.

N = the number of people

100 + 5n

Connecting writing expressions with story problems and/or drawing pictures will give students a context for this work. It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number.

Example 1

Maria has three more than twice as many crayons as Elizabeth. Write an algebraic expression to represent the number of crayons that Maria has.

Solution: 2c + 3 where c represents the number of crayons that Elizabeth has.

Example 2

An amusement park charges $28 to enter and $0.35 per ticket. Write an algebraic expression to represent the total amount spent.

Solution: 28 + 0.35t where t represents the number of tickets purchased

Example 3

Andrew has a summer job doing yard work. He is paid $15 per hour and a $20 bonus when he completes the yard. He was paid $85 for completing one yard.

Write an equation to represent the amount of money he earned.

Solution: 15h + 20 = 85 where h is the number of hours worked

Example 4

Describe a problem situation that can be solved using the equation 2c + 3 = 15; where c represents the cost of an item

Example 5

Bill earned $5.00 mowing the lawn on Saturday. He earned more money on Sunday. Write an expression that shows the amount of money Bill has earned.

Solution: $5.00 + n

Common Misconceptions / Challenges

Students confuse expression and equation. When writing an expression, they may include an equal sign when they should not

Students may confuse the operations associated with certain vocabulary words.

Criteria for Success (Performance Level Descriptors)

Limited: Understand the use of variables in simple mathematical expressions
Basic: Understand the use of variables in simple mathematical expressions
Proficient: Understand the use of variables in simple mathematical expressions
Accelerated: N/A
Advanced: N/A

Prior Knowledge	**Future Learning**
5.OA.A.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.	7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form 7.EE.4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities

Career Connections

- Accountants
- Counter and rental clerks
- Engineers (all types)
- Doctors / Nurses
- Order processors
- Electricians
- Mechanics

Ohio's Learning Standards- Clear Learning Targets
Math, Grade 6

6.EE.7	Essential Understanding	Vocabulary
6.EE.7 Solve real-word and mathematical problems by writing and solving equations of the form *x* + *p* = *q* and *px* =*q* for cases in which *p*, *q* and *x* are all nonnegative rational numbers.	Calculate the solution of one-step equations by using knowledge of order of operations and the properties of equality for addition, subtraction, multiplication, and division.	• Operation • inverse operations • subtraction property of equality • rational number • nonnegative • solve • algebraic equation

Essential Skills

- I can define an inverse operation.
- I can solve equations in the form x + p = q are given numbers
- I can solve equations in the form px = q where p and q are given numbers
- I can write and solve algebraic equations that represent real world problems

Instructional Methods

Students have used algebraic expressions to generate answers given values for the variable. This understanding is now expanded to equations where the value of the variable is unknown but the outcome is known. For example, in the expression, x + 4, any value can be substituted for the x to generate a numerical answer; however, in the equation x + 4 = 6, there is only one value that can be used to get a 6. Problems should be in context when possible and use only one variable.

Students write equations from real-world problems and then use inverse operations to solve one-step equations. Equations may include fractions and decimals with non-negative solutions.

Students create and solve equations that are based on real world situations. It may be beneficial for students to draw pictures that illustrate the equation in problem situations. Solving equations using reasoning and prior knowledge should be required of students to allow them to develop effective strategies.

Example 1

Meagan spent $56.58 on three pairs of jeans. If each pair of jeans costs the same amount, write an algebraic equation that represents this situation and solve to determine how much one pair of jeans cost.

Solution

Sample: Students might say: “I created the bar model to show the cost of the three pairs of jeans. Each bar labeled J is the same size because each pair of jeans costs the same amount of money. The bar model represents the equation 3J = $56.58. To solve the problem, I need to divide the total cost of 56.58 between the three pairs of jeans. I know that it will be more than $10 each because 10 x 3 is only 30 but less than $20 each because 20 x 3 is 60. If I start with $15 each, I am up to $45. I have $11.58 left. I then give each pair of jeans $3. That’s $9 more dollars. I only have $2.58 left. I continue until all the money is divided. I ended up giving each pair of jeans another $0.86. Each pair of jeans costs $18.86 (15+3+0.86). I double check that the jeans cost $18.86 each because $18.86 x 3 is $56.58.”

$56.58		
J	J	J

Example 2

Julio gets paid $20 for babysitting. He spends $1.99 on a package of trading cards and $6.50 on lunch. Write and solve an equation to show how much money Julio has left.

20		
1.99	6.50	money left over (m)

Solution: 20 = 1.99 + 6.50 + x, x = $11.51

Common Misconceptions / Challenges

Students may not apply the inverse operation when solving equations. Remind students to pay close attention to the sign in the equation and check their answers at the end by substituting it back in for the variable and seeing if they get a true number equation.

Students may have trouble with equations when the variable is on the right side of the equal sign. Demonstrate that they can rewrite the equation with the variable on the left side.

Students may not understand that some real life situations require them to combine like terms before they solve for the variable.

Criteria for Success (Performance Level Descriptors)

- **Limited:** N/A
- **Basic:** Solve one-step equations with positive integer coefficients
- **Proficient:** Write and solve one-step equations with positive integer coefficients
- **Accelerated:** Write and solve equations with positive rational coefficients
- **Advanced:** N/A

Prior Knowledge

5.OA.A.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

Future Learning

7.EE.4.A Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and rare specific rational numbers.

Career Connections

- Accountants
- Counter and rental clerks
- Medical equipment maker
- Doctors / Nurses
- Order processors
- Electricians
- Wood workers
- Billing support

Ohio's Learning Standards- Clear Learning Targets

Math, Grade 6

6.EE.8	Essential Understanding	Vocabulary
6.EE.8 Write an inequality of the form $x > c$ *or* $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	Students recognize that inequalities of the form $x < c$ and $x > c$, where x is a variable and c is a fixed number have infinitely many solutions when the values of x come from a set of rational numbers.	• greater than • less than • inequality • solution

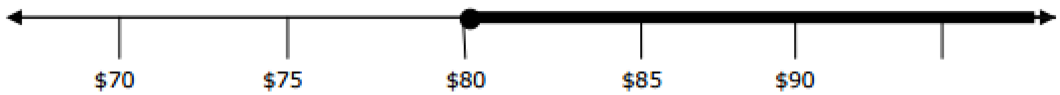
Essential Skills

- I can identify the constraint or condition in a real-world or mathematical problem in order to set up an inequality.
- I can recognize that inequalities of the form x>c or x<c have infinitely many solutions.
- I can explain what the solution set of an inequality represents
- I can write an inequality of the form x>c or x<c to represent a constraint or condition in a real-world or mathematical problem.
- I can represent solutions to inequalities on the number line diagrams.

Instructional Methods

6.EE.8

Many real-world situations are represented by inequalities. Students write an inequality and represent solutions on a number line for various contextual situations. For example, the class must raise at least $80 to go on the field trip. If m represents money, then the inequality m ≥ to $80. Students recognize that possible values can include too many decimal values to name. Therefore, the values are represented on a number line by shading.

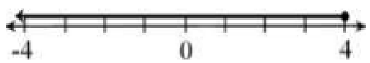


A number line diagram is drawn with an open circle when an inequality contains a < or > symbol to show solutions that are less than or greater than the number but not equal to the number. The circle is shaded, as in the example above, when the number is to be included. Students recognize that possible values can include fractions and decimals, which are represented on the number line by shading. Shading is extended through the arrow on a number line to show that an inequality has an infinite number of solutions.

Example 1

Graph $x \leq 4$

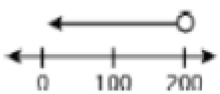
Solution



Example 2

The Flores family spent less than $200.00 last month on groceries. Write an inequality to represent this amount and graph this inequality on a number line.

Solution:



$200 > x$, where x is the amount spent on groceries.

Common Misconceptions / Challenges Students may not recognize that a number is a solution of an inequality when the symbol ≤ or ≥ is used and the two numbers on either side of the symbol are equal. Students may have difficulty understanding which inequality symbol to use with phrases such as “at least” or “no more than” Students often use an open or closed dot incorrectly when graphing an inequality.
Criteria for Success (Performance Level Descriptors) - **Limited:** N/A - **Basic:** N/A - **Proficient:** Write and graph solutions to inequalities on a number line - **Accelerated:** Given a situation, write an inequality and graph solutions on a number line - **Advanced:** Given a complex situation, write an inequality and graph solutions on a number line

Prior Knowledge	**Future Learning**
4.NBT.A.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	7.EE.4.b Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and rare specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

Career Connections

Many jobs are out there that require the ability to solve and graph inequalities. Careers include:

- Accountants
- Counter and rental clerks
- Medical equipment maker
- Doctors / Nurses
- Order processors
- Electricians
- Wood workers
- Billing support

Ohio's Learning Standards- Clear Learning Targets

Math, Grade 6

6.EE.9

Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

Essential Understanding

Students should be able to create a table by placing the independent variable in the first row or column and the dependent variable in the second row or column. They compute entries in the table by choosing arbitrary values for the independent variable (no constraints) and then determine what the dependent variable must be.

Vocabulary

- variable
- independent variable
- dependent variable
- coordinate plane

Essential Skills

- I can create a table of two variables that represents a real-world situation in which one quantity will change in relation to the other
- I can explain the difference between the independent variable and the dependent variable and give examples of both
- I can write an algebraic equation that represents the relationship between the two variables
- I can create a graph by plotting the dependent variable on the x-axis and the independent variable on the y-axis of a coordinate plane
- I can analyze the relationship between the dependent and independent variables by comparing the table, graph and equation

Instructional Strategies

The purpose of this standard is for students to understand the relationship between two variables, which begins with the distinction between dependent and independent variables. The independent variable is the variable that can be changed; the dependent variable is the variable that is affected by the change in the independent variable. Students recognize that the independent variable is graphed on the x-axis; the dependent variable is graphed on the y-axis.

Students recognize that not all data should be graphed with a line. Data that is discrete would be graphed with coordinates only. Discrete data is data that would not be represented with fractional parts such as people, tents, records, etc. For example, a graph illustrating the cost per person would be graphed with points since part of a person would not be considered. A line is drawn when both variables could be represented with fractional parts.

Students are expected to recognize and explain the impact on the dependent variable when the independent variable changes (As the x variable increases, how does the y variable change?) Relationships should be proportional with the line passing through the origin. Additionally, students should be able to write an equation from a word problem and understand how the coefficient of the dependent variable is related to the graph and/or a table of values.

Students can use many forms to represent relationships between quantities. Multiple representations include describing the relationship using language, a table, an equation, or a graph. Translating between multiple representations helps students understand that each form represents the same relationship and provides a different perspective.

Example 1

What is the relationship between the two variables? Write an expression that illustrates the relationship.

x	1	2	3	4
y	2.5	5	7.5	10

Solution: y = 2.5x

Example 2

Susan started with $1 in her savings. She plans to add $4 per week to her savings. Use an equation, table and graph to demonstrate the relationship between

the number of weeks that pass and the amount in her savings account.

Solution

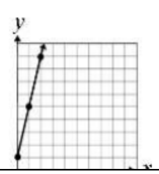
Language: Susan has $1 in her savings account. She is going to save $4 each week.

Equation: y = 4x + 1

Table

x	y
0	1
1	5

Graph



Common Misconceptions / Challenges

Students may write a rule based on the first input and output only. Tell them to make sure that their rule works for all numbers in the table.

When graphing a linear function, students may only graph two ordered pairs. Since any two points will make a line, have students graph at least 3 ordered pairs to check their work.

Criteria for Success (Performance Level Descriptors)

Limited: Write a one-variable equation to express one quantity in terms of the other quantity

Basic: Write a one-variable equation to express one quantity in terms of the other quantity

Proficient: Write a one-variable equation to express one quantity in terms of the other quantity

Accelerated: Use tables and graphs to analyze the relationship between dependent and independent variables and relate these to the equation

Advanced: Analyze the relationship between dependent and independent variables represented in tables and graphs, and then write an appropriate equation

Prior Knowledge	Future Learning
5.OA.B.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms.	8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

Career Connections

Many careers depend on the employee's ability to work with variables. These include:

- Engineers
- Researchers
- Physicians
- Chemists
- Geologists
- Weather forecasters
- Teachers
- Attorneys

Ohio's Learning Standards- Clear Learning Targets

Math, Grade 6

6.G.1 G.1 Through composition into rectangles or decomposition into triangles, find the area of right triangles, other triangles, special quadrilaterals, and polygons; apply these techniques in the context of solving real-world and mathematical problems.	**Essential Understanding** Students will discover that irregular shapes can often be decomposed, or cut into, smaller shapes. Doing this can help students determine the area of the irregular shape.	**Vocabulary** - area - triangle - quadrilateral - right triangle - parallelogram - polygon - trapezoid

Essential Skills

- I can recognize and know how to compose and decompose polygons into triangles and rectangles.
- I can compare the area of a triangle to the area of the composed rectangle.
- I can apply the techniques of composing and/or decomposing to find the area of triangles, special quadrilaterals and polygons to solve mathematical and real world problems.

Instructional Methods

Students continue to understand that area is the number of squares needed to cover a plane figure. Finding the area of triangles is introduced in relationship to the area of rectangles – a rectangle can be decomposed into two congruent triangles. Therefore, the area of the triangle is ½ the area of the rectangle. The area of a rectangle can be found by multiplying base x height; therefore, the area of the triangle is ½ bh or (b x h)/2. Students decompose shapes into rectangles and triangles to determine the area. For example, a trapezoid can be decomposed into triangles and rectangles Using the trapezoid's dimensions, the area of the individual triangle(s) and rectangle can be found and then added together.

Students should know the formulas for rectangles and triangles. "Knowing the formula" does not mean memorization of the formula. To "know" means to have an understanding of why the formula works and how the formula relates to the measure (area) and the figure. Special quadrilaterals include rectangles, squares, parallelograms, trapezoids, rhombi, and kites.

Example 1

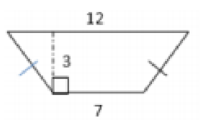
Find the area of a triangle with a base length of three units and a height of four units.

Solution:

A = ½ bh
A = ½ (3 units)(4 units)
A = ½ 12 units^2
A = 6 units^2

Example 2

Find the area of the trapezoid shown below using the formulas for rectangles and triangles.



Solution: The trapezoid could be decomposed into a rectangle with a length of 7 units and a height of 3 units. The area of the rectangle would be 21 units2. The triangles on each side would have the same area. The height of the triangles is 3 units. After taking away the middle rectangle's base length, there is a total of 5 units remaining for both of the side triangles. The base length of each triangle is half of 5. The base of each triangle is 2.5 units. The area of one triangle would be 1⁄2 (2.5 units)(3 units) or 3.75 units ^{2} (squared).

21 units^2
3.75 units^2
+3.75 units^2
28.5 units^2

Example 3

A rectangle measures 3 inches by 4 inches. If the lengths of each side double, what is the effect on the area?

Solution: The new rectangle would have side lengths of 6 inches and 8 inches. The area of the original rectangle was 12 in^2. The area of the new rectangle is 48 in^2. The area increased 4 times (quadrupled). Students may also create a drawing to show this visually.

Example 4

The area of the rectangular school garden is 24 square units. The length of the garden is 8 units. What is the length of the fence needed to enclose the entire garden?

Solution:

First students need to find the width of the garden

A= l x w

24=8 x w

24 ÷8 = w

W= 3

Then, they need to understand that fencing deals with perimeter, so the perimeter is 2l+2w

2(8) + 2(3)= 22 units of fence

Example 5

The lengths of the sides of a bulletin board are 4 feet by 3 feet. How many index cards measuring 4 inches by 6 inches would be needed to cover the board?

Solution:

Change the dimensions of the bulletin board to inches. The area of the board would be 48 inches x 36 inches or 1728 in^2. The area of one index care is 12 in^2. Divide 1728 by 24 to get the number of index cards, which is 72.

Common Misconceptions / Challenges

Students are sometimes unable to find the height of a triangle because sometimes it is inside the triangle, sometimes it is the side of the triangle and sometimes it is outside the triangle. Stress the importance that the height is always from the top vertex straight down to the base, creating a right angle.

Remind students to always include units when writing the measurement, even if the unit of measurement is units^2

Students often forget to divide by 2 (or multiply by ½) when finding the area of a triangle. Remind them that the triangle is half of a parallelogram.

With trapezoids, remind students that the two bases are always parallel to each other and perpendicular to the height.

Criteria for Success (Performance Level Descriptors)

Limited: Find areas of right triangles using grid paper

Basic: Find areas of polygons with whole number side lengths by decomposing them into rectangles and triangles

Proficient: Find areas of polygons by decomposing them into rectangles and triangles

Accelerated: Solve mathematical problems by finding the area of a two-dimensional shape composed of rectangles and triangles

Advanced: Solve non-routine mathematical problems by finding the area of a two-dimensional shape composed of rectangles and triangles

Prior Knowledge

5.G.3. Identify and describe commonalities and differences of triangles based on angle measures (equiangular, right, acute, and obtuse triangles) and side lengths (isosceles, equilateral, and scalene triangles).

5.G 4. Identify and describe commonalities and differences of quadrilaterals based on angle measures, side lengths, and the presence or absence of parallel and perpendicular lines, e.g., squares, rectangles, parallelograms, trapezoids, and rhombuses.

Future Learning

7.G.1. Solve problems involving similar figures with right triangles, other triangles, and special quadrilaterals

7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Career Connections

Careers using geometry, area, and decomposing shapes include:

- Computer systems manager
- Construction manager
- Food services manager
- Chemists and material scientists
- Counter and rental clerks
- Real estate brokers

Ohio's Learning Standards- Clear Learning Targets

Math, Grade 6

6.G.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = B h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	**Essential Understanding** Apply the formula V = lwh to find the volume of a right rectangular prism and use the correct volume units when writing the answer. Students extend the volume formula for a right rectangular prism to the formula V = Area of base times height. They understand that any face can be the base. -	**Vocabulary** - volume - right rectangular prism - edge - base - height - area - cubic unit

Essential Skills

- I can calculate the volume of a right rectangular prism.
- I can apply volume formulas for right rectangular prisms to solve real-world and mathematical problems involving rectangular prisms with fractional edge lengths.
- I can model the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths.

Instructional Strategies

Previously students calculated the volume of right rectangular prisms (boxes) using whole number edges. The unit cube was 1 x 1 x 1. In 6th grade the unit cube will have fractional edge lengths. (ie. 1/2 · 1/2 · 1/2) Students find the volume of the right rectangular prism with these unit cubes.

"Know the formula" does not mean memorization of the formula. To "know" means to have an understanding of why the formula works and how the formula relates to the measure (volume) and the figure.

Students need multiple opportunities to measure volume by filling rectangular prisms with blocks and looking at the relationship between the total volume and the area of the base. Through these experiences, students derive the volume formula (volume equals the area of the base times the height). Students can explore the connection between filling a box with unit cubes and the volume formula using interactive applets. In addition to filling boxes, students can draw diagrams to represent fractional side lengths, connecting with multiplication of fractions. This process is similar to composing and decomposing two dimensional shapes.

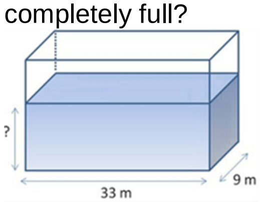
Example 1

Find the volume, in cubic feet, of the right rectangular prism shown below.

Solution: 500 ft^3

Example 2

A swimming pool is 33m by 9m. the swimming pool water volume when completely full is 2,376 cubic m. What is the height of the water when the pool is completely full?



Solution: 8 m

Common Misconceptions

Students often forget that the appropriate units for volume is cubic units and that it is a linear unit when finding the length, width or hight of the prism.

Criteria for Success (Performance Level Descriptors)

Limited: Find the volume of rectangular prisms with whole number sides

Basic: Find volumes of rectangular prisms with whole number side lengths

Proficient: Find volumes of rectangular prisms with fractional edge lengths

Accelerated: Solve mathematical problems by finding the volumes of rectangular prisms with fractional edge lengths

Advanced: Solve real-world problems by finding the volumes of rectangular prisms with fractional edge lengths

Prior Knowledge

5. MD.3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement

5.MD.4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

5.MD.5. Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.

Future Learning

7.G.6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms

Career Connections

Careers using geometry and rectangular prisms include:

- Landscape architect
- Graphic designer
- Photographer
- Physicist
- Astronomer
- Artist
- Pilot

Ohio's Learning Standards- Clear Learning Targets
Math, Grade 6

6.G.3	Essential Understanding	Vocabulary
Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	Given coordinates for the vertices, students draw polygons in the coordinate plane. Students find the area enclosed by a polygon by composing or decomposing using polygons with known area formulas.	- polygon - coordinate - coordinate plane - vertices/vertex

Essential Skills

- I can plot vertices in the coordinate plane to draw specific polygons
- I can use the coordinates of the vertices of a polygon to find the length of a specific side
- I can apply the technique of using coordinates to find the length of a side of a polygon drawn in the coordinate plane to solve real- world and mathematical problems.

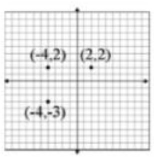
Instructional Strategies

Students are given the coordinates of polygons to draw in the coordinate plane. If both x-coordinates are the same (2, - 1) and (2, 4), then students recognize that a vertical line has been created and the distance between these coordinates is the distance between -1 and 4, or 5. If both the y-coordinates are the same (-5, 4) and (2, 4), then students recognize that a horizontal line has been created and the distance between these coordinates is the distance between -5 and 2, or 7. Using this understanding, student solve real-world and mathematical problems, including finding the area of quadrilaterals and triangles.

This standard can be taught in conjunction with Grade6.G.1 to help students develop the formula for the triangle by using the squares of the coordinate grid. Given a triangle, students can make the corresponding square or rectangle and realize the triangle is 1 2.

Students' progress from counting the squares to making a rectangle and recognizing the triangle as ½ leading to the development of the formula for the area of a triangle.

Example 1
If the points on the coordinate plane below are the three vertices of a rectangle, what are the coordinates of the fourth vertex? How do you know? What are the length and width of the rectangle? Find the area and the perimeter of the rectangle.



Solution
The fourth vertex would be (2,-3)
The area would be 5x6 or 30 units^2
The perimeter would be 5+5+6+6 or 22 units^2

Example 2
On a map, the library is located at (-2, 2), the city hall building is located at (0,2), and the high school is located at (0,0). Represent the locations as points on a coordinate grid with a unit of 1 mile.

- What is the distance from the library to the city hall building?
- What is the distance from the city hall building to the high school? How do you know?
- What shape is formed by connecting the three locations?
- The city council is planning to place a city park in this area. How large is the area of the planned park?

Solution
The distance from library to city hall is 2 miles.

The distance from city hall to the high school is 2 miles. I know this because they have the same x coordinate and the distance between 2 and 0 is 2.

The three locations form a right triangle. The area is 2 mi^2.

Common Misconceptions / Challenges

Students can still be confused at which comes first, the horizontal movement or vertical movement, when plotting a point. Remind them that x comes before y alphabetically, so the x movement comes before the y movement. The x movement goes from left to right, just as number lines did that they learned prior to this.

Some students forget to start the origin when plotting or reading points. Sometimes, they use an ordered pair as an amount to move away from a previously-plotted point. Remind them to always refer back to the origin.

Criteria for Success (Performance Level Descriptors)

Limited:

Basic: Draw polygons in one quadrant of the coordinate plane

Proficient: Draw polygons in the coordinate plane

Accelerated: Draw polygons in the coordinate plane and find lengths of horizontal and vertical sides to solve real-world problems

Advanced: Solve real-world and mathematical problems by graphing points and/or polygons in the coordinate plane

Prior Knowledge	**Future Learning**
5.G.1. Define and understand parts of a coordinate plane and how to plot points 5.G.2. Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	8.G.8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system

Career Connections

Careers using geometry, graphing, and coordinate pairs:

- Map making
- Surveyors
- Telecommunications
- Airline Pilots
- Sales managers
- Architect

Ohio's Learning Standards- Clear Learning Targets
Math, Grade 6

6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	**Essential Understanding** Figures can be 'taken apart' or 'cut apart' into flat pieces called nets. Nets are used to determine the surface area of three-dimensional figures.	**Vocabulary** - net - surface area - right rectangular prism - right triangular prism

Essential Skills
- I can recognize that 3-D figures can be represented by nets.
- I can represent three-dimensional figures using nets made up of rectangles and triangles.
- I can apply knowledge of calculating the area of rectangles and triangles to a net.
- I can combine the areas for rectangles and triangles in the net to find the surface area of a 3-dimensional figure.
- I can solve real-world and mathematical problems involving surface area using nets.

Instructional Strategies

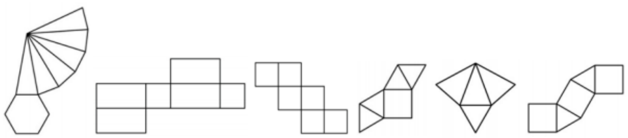
A net is a two-dimensional representation of a three-dimensional figure. Students represent three- dimensional figures whose nets are composed of rectangles and triangles. Students recognize that parallel lines on a net are congruent. Using the dimensions of the individual faces, students calculate the area of each rectangle and/or triangle and add these sums together to find the surface area of the figure.

Students construct models and nets of three-dimensional figures, describing them by the number of edges, vertices, and faces. Solids include rectangular and triangular prisms. Students are expected to use the net to calculate the surface area.

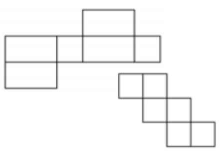
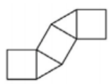
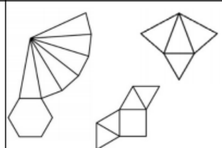
Students also describe the types of faces needed to create a three-dimensional figure. Students make and test conjectures by determining what is needed to create a specific three-dimensional figure.

Example 1

Classify each net as representing a rectangular prism, a triangular prism, or a pyramid.

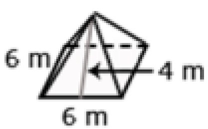


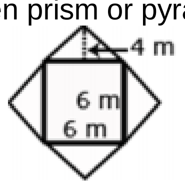
Solution

Nets Forming a Rectangular Prism	Nets Forming a Triangular Prism	Nets Forming a Pyramid
		

Example 2

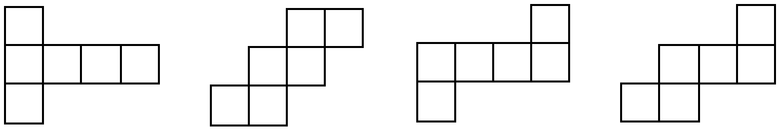
Create the net for a given prism or pyramid, and then use the net to calculate the surface area.





Common Misconceptions / Challenges

Understanding that there are multiple nets for the same object may be difficult for some to visualize, provide concrete examples of nets for the object. Both the composition and decomposition of rectangular prisms should be explored. The understanding that there may be multiple nets that create a cube may be challenging. For example the following are a few of the possible nets that will create a cube.



Stuents may accidentally leave out the area of a face when calculating surface area. Remind them that rectangular prisms and cubes have six faces, all of which should be added to find the surface area.

Criteria for Success (Performance Level Descriptors)

Limited:

Basic: Given nets, find surface areas of rectangular prisms with whole number side lengths

Proficient: Use nets to find surface areas of rectangular and triangular prisms and pyramids

Accelerated: N/A

Advanced: N/A

Prior Knowledge	Future Learning
5.G.3 Classify two-dimensional figures into categories based on their properties.	7.G.3. Describe the two-dimensional figures that result from slicing three dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids

Career Connections

- Landscaper
- Architect
- Engineer
- Astronomer
- Electrician
- Surveyor

Ohio's Learning Standards- Clear Learning Targets

Math, Grade 6

6.SP.1

6.SP.1 Develop statistical reasoning by using the GAISE model: a. Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because of the variability in students' ages. (GAISE Model, step 1) b. Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2) c. Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3) d. Interpret Results: Draw logical conclusions from the data based on the original question. (GAISE Model, step 4)

Essential Understanding

Students should be able to distinguish between questions that are statistical and those that are not statistical. They understand that statistical questions involve variability.

Vocabulary

- Variability
- Statistical
- Quantitative
- Data

Essential Skills

- I can explain what makes a good statistical question
- I can develop a question that can be used to collect statistical information

Instructional Strategies

Students differentiate between statistical questions and those that are not. A statistical question is one that collects information that addresses differences in a population. The question is framed so that the responses will allow for the differences. For example, the question, “How tall am I?” is not a statistical question because there is only one response; however, the question, “How tall are the students in my class?” is a statistical question since the responses would allow for differences.

Statistics are numerical data relating to an aggregate of individuals; statistics is also the name for the science of collecting, analyzing and interpreting such data. A statistical question anticipates an answer that varies from one individual to the next and is written to account for the variability in the data. Data are the numbers produced in response to a statistical question. Data are frequently collected from surveys or other sources (i.e. documents).

Questions can result in a narrow or wide range of numerical values. For example, asking classmates “How old are the students in my class in years?” will result in less variability than asking “How old are the students in my class in months?”

Students might want to know about the fitness of the students at their school. Specifically, they want to know about the exercise habits of the students. So rather than asking "Do you exercise?" they should ask about the amount of exercise the students at their school get per week. A statistical question for this study could be: “How many hours per week on average do students at Jefferson Middle School exercise?”

To collect this information, students might design a survey question that anticipates variability by providing a variety of possible anticipated responses that have numerical answers, such as: 3 hours per week, 4 hours per week, and so on. Be sure that students ask questions that have specific numerical answers

Common Misconceptions / Challenges

Students may have a difficult time creating statistical questions that will demonstrate variability in numerical data. All questions they ask may be categorical data.

Criteria for Success (Performance Level Descriptors)

6.SP.1

Limited: N/A
Basic: N/A
Proficient: N/A
Accelerated: N/A
Advanced: N/A

Prior Knowledge	Future Learning
	7.SP.1. Understand that statistics can be used to gain information about a population by examining a sample of the population. 7.SP.2. Broaden statistical reasoning by using the GAISE model

Career Connections

Market research

Meteorology

Actuarial Science

Public Health

Ohio's Learning Standards- Clear Learning Targets

Math, Grade 6

6.SP.2-3

2: Understand that a set of data collected to answer a statistical question has a distribution, which can be described by its center, spread, and overall shape.

3: Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Essential Understanding

Data collected to answer a statistical question has a distribution that is often summarized in terms of center, variability, and shape

Vocabulary

- variability
- set
- data
- distribution
- center
- spread
- shape
- numerical data set
- measure of center
- mean
- median
- mode
- measure of variation
- range
- interquartile range
- extremes
- upper/lower quartile
- outlier
- mean absolute deviation

Essential Skills

- I can identify that a set of data has distribution.
- I can describe a set of data by its center, e.g., mean and median.
- I can describe a set of data by its spread and overall shape, e.g. by identifying data clusters, peaks, gaps and symmetry.
- I can recognize there are measures of central tendency for a data set, e.g., mean, median, mode.
- I can recognize there are measures of variances for a data set, e.g., range, interquartile range, mean absolute deviation.
- I can recognize that measure of central tendency for a data set summarizes the data with a single number.

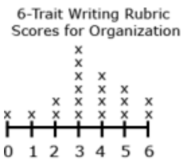
Instructional Strategies

6.SP.2

The distribution is the arrangement of the values of a data set. Distribution can be described using center (median or mean), and spread. Data collected can be represented on graphs, which will show the shape of the distribution of the data. Students examine the distribution of a data set and discuss the center, spread, and overall shape with dot plots, histograms, and box plots.

Example 1

The dot plot shows the writing scores for a group of students on organization. Describe the data.



Solution:

The values range from 0 – 6. There is a peak at 3. The median is 3, which means 50% of the scores are greater than or equal to 3 and 50% are less than or equal to 3. The mean is 3.68. If all students scored the same, the score would be 3.68

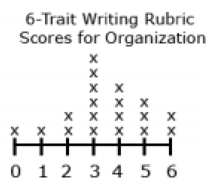
6.SP.3

Data sets contain many numerical values that can be summarized by one number such as a measure of center. The measure of center gives a numerical value to represent the center of the data (ie. midpoint of an ordered list or the balancing point). Another characteristic of a data set is the variability (or spread) of the values. Measures of variability are used to describe this characteristic.

Example 1

Consider the data shown in the dot plot of the six trait scores for organization for a group of students.

- How many students are represented in the data set?
- What are the mean and median of the data set? What do these values mean? How do they compare?
- What is the range of the data? What does this value mean?



Solution

- 19 students are represented in the data set
- The mean of the data set is 3.5. The median is 3. The mean indicates that is the values were equally distributed, all students would score a 3.5. The median indicates that 50% of the students scored a 3 or higher; 50% of the students scored a 3 or lower.
- The range of the data is 6, indicating that the values vary 6 points between the lowest and highest scores.

Common Misconceptions / Challenges

When finding the mean, students may not include all of the data values in a dot plot, especially when the values repreat. Remind them that all numbers must be included.

Students may not understand the importance of writing the data in numerical order when calculating the median.

Students may miscalculate the first quartile or third quartile when there is an even number of data values in the first and third halves of the data. Point out that when there are two middle numbers in the lower half or the upper half, the first or third quartile is the mean of those two numbers.

When calculating the mean absolute deviation, students may incorrectly find the differences between each value in the dtga set and the mean rather than finding the absolute value of the differences.

Criteria for Success (Performance Level Descriptors)

6.SP.2

Limited: N/A
Basic: N/A
Proficient: N/A
Accelerated: N/A
Advanced: N/A

6.SP.3

Limited: N/A
Basic: N/A
Proficient: N/A
Accelerated: N/A
Advanced: N/A

Prior Knowledge

4.MD.4. Display and interpret data in graphs (picture graphs, bar graphs, and line plots) to solve problems using numbers and operations for this grade

Future Learning

7.SP. 3. Describe and analyze distributions

Career Connections

mathematician

math teacher

statistical analyst

psychologist

accountant

Ohio's Learning Standards- Clear Learning Targets
Math, Grade 6

6.SP.4-5

6.SP.4 Display numerical data in plots on a number line, including dot plots (line plots), histograms, and box plots. (GAISE Model, step 3)

6.SP.5 Summarize numerical data sets in relation to their context.

a. Report the number of observations.
b. Describe the nature of the attribute under investigation, including how it was measured and its units of measurement.
c. Find the quantitative measures of center (median and/or mean) for a numerical data set and recognize that this value summarizes the data set with a single number. Interpret mean as an equal or fair share. Find measures of variability (range and interquartile range) as well as informally describe the shape and the presence of clusters, gaps, peaks, and outliers in a distribution.
d. Choose the measures of center and variability, based on the shape of the data distribution and the context in which the data were gathered.

Essential Understanding

Data can be represented in many different graphical ways.

Median is a measure of center and *interquartile range* is a measure of variability. Students learn that these measures are preferred when the shape of the distribution is skewed.

Vocabulary

- Line plot
- Dot plot
- Histogram
- Median
- Lower extreme
- Lower quartile
- Upper quartile
- Upper extreme
- Box plot
- Outlier
- Measure of center
- Interquartile range
- Mean absolute deviation

Essential Skills

- I can identify the components of dot plots, histograms, and box plots.
- I can find the median, quartile and interquartile range of a set of data.
- I can analyze a set of data to determine its variance.
- I can create a dot plot to display a set of numerical data.
- I can report the number of observations in a data set or display.
- I can organize and display data in tables and graphs.
- I can calculate quantitative measures of center.
- I can identify outliers.
- I can determine the effect of outliers on quantitative measures of a set of data, e.g., mean, median, mode, range, interquartile range, mean absolute deviation.

Instructional Strategies
SP.4

Students display data graphically using number lines. Dot plots, line plot, histograms, and box plots are four graphs to be used. Students are expected to determine the appropriate graph as well as read data from graphs generated by others.

Dot plots are simple plots on a number line where each dot represents a piece of data in the data set. Dot plots are suitable for small to moderate size data sets and are useful for highlighting the distribution of the data including clusters, gaps, and outliers.

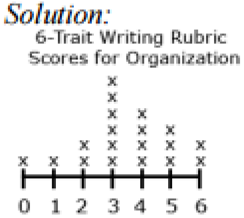
A histogram shows the distribution of continuous data using intervals on the number line. The height of each bar represents the number of data values in that interval. In most real data sets, there is a large amount of data and many numbers will be unique. A graph (such as a dot plot) that shows how many ones, how many twos, etc. would not be meaningful; however, a histogram can be used. Students group the data into convenient ranges and use these intervals to generate a frequency table and histogram. Note that changing the size of the bin changes the appearance of the graph and the conclusions may vary from it.

A box plot shows the distribution of values in a data set by dividing the set into quartiles. It can be graphed either vertically or horizontally. The box plot is constructed from the five-number summary (minimum, lower quartile, median, upper quartile, and maximum). These values give a summary of the shape of a distribution. Students understand that the size of the box or whiskers represents the middle 50% of the data.

Example 1

Nineteen students completed a writing sample that was scored on organization. The scores for organization were 0, 1, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6. Create a data display. What are some observations that can be made from the data display?

Solution:

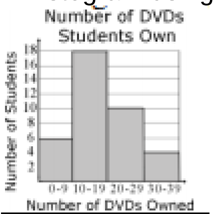


Example 2:

Grade 6 students were collecting data for a math class project. They decided they would survey the other two grade 6 classes to determine how many DVDs each student owns. A total of 48 students were surveyed. The data are shown in the table below in no specific order. Create a data display. What are some observations that can be made from the data display?

11	21	5	12	10	31	19	13	23	33
10	11	25	14	34	15	14	29	8	5
22	26	23	12	27	4	25	15	7	
2	19	12	39	17	16	15	28	16	

Solution:
A histogram using 5 intervals (0-9, 10-19 …30-39) to organize the data is displayed below



Most of the students have between 10 and 19 DVDs as indicated by the peak on the graph. The data is pulled to the right since only a few students own more than 30 DVDs.

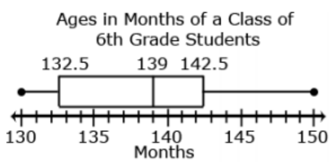
Example 3:

Ms. Wheeler asked each student in her class to write their age in months on a sticky note. The 28 students in the class brought their sticky note to the front of the room and posted them in order on the white board. The data set is listed below in order from least to greatest. Create a data display. What are some observations that can be made from the data display?

130	130	131	131	132	132	132	133	134	136
137	137	138	139	139	139	140	141	142	142
142	143	143	144	145	147	149	150		

Solution

Five number summary
Minimum – 130 months
Quartile 1 (Q1) – (132 + 133) ÷ 2 = 132.5 months
Median (Q2) – 139 months
Quartile 3 (Q3) – (142 + 143) ÷ 2 = 142.5 months
Maximum – 150 months



This box plot shows that

- ¼ of the students in the class are from 130 to 132.5 months old
- ¼ of the students in the class are from 142.5 months to 150 months old
- ½ of the class are from 132.5 to 142.5 months old
- The median class age is 139 months

SP.5
Students summarize numerical data by providing background information about the attribute being measured, methods and unit of measurement, the context of data collection activities (addressing random sampling), the number of observations, and summary statistics. Summary statistics include quantitative measures of center (median and median) and variability (interquartile range and mean absolute deviation) including extreme values (minimum and maximum), mean, median, mode, range, and quartiles.

Students should record the number of observations. Using histograms, students determine the number of values between specified intervals. Given a box plot and the total number of data values, students identify the number of data points that are represented by the box. Reporting of the number of observations must consider the attribute of the data sets, including units (when applicable).

Measures of center given a set of data values, students summarize the measure of center with the median or mean. The median is the value in the middle of an ordered list of data. This value means that 50% of the data is greater than or equal to it and that 50% of the data is less than or equal to it.

The mean is the arithmetic average; the sum of the values in a data set divided by how many values there are in the data set. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students develop these understandings of what the mean represents by redistributing data sets to be level or fair (equal distribution) and by observing that the total distance of the data values above the mean is equal to the total distance of the data values below the mean (balancing point).

Students use the concept of mean to solve problems. Given a data set represented in a frequency table, students calculate the mean. Students find a missing value in a data set to produce a specific average.

Measures of Variability Measures of variability/variation can be described using the interquartile range or the Mean Absolute Deviation. The interquartile range (IQR) describes the variability between the middle 50% of a data set. It is found by subtracting the lower quartile from the upper quartile. It represents the length of the box in a box plot and is not affected by outliers. Students find the IQR from a data set by finding the upper and lower quartiles and taking the difference from reading a box plot.

Mean Absolute Deviation (MAD) describes the variability of the data set by determining the absolute deviation (the distance) of each data piece from the mean and then finding the average of these deviations. Both the interquartile range and the Mean Absolute Deviation are represented by a single numerical value. Higher values represent a greater variability in the data.

Students describe the context of the data, using the shape of the data, and are able to use this information to determine an appropriate measure of center and measure of variability. The measure of center that a student chooses to describe a data set will depend upon the shape of the data distribution and context of data collection. The mode is the value in the data set that occurs most frequently. The mode is the least frequently used as a measure of center because data sets may not have a mode, may have more than one mode, or the mode may not be descriptive of the data set. The mean is a very common measure of center computed by adding all the numbers in the set and dividing by the number of values. The mean can be affected greatly by a few data points that are very low or very high. In this case, the median or middle value of the data set might be more descriptive. In data sets that are symmetrically distributed, the mean and median will be very close to the same. In data sets that are skewed, the mean and median will be different, with the median frequently providing a better overall description of the data set.

Example 1

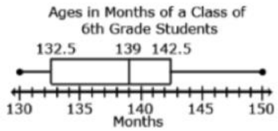
Susan has four 20-point projects for math class. Susan's scores on the first 3 projects are shown below: Project 1: 18 Project 2: 15 Project 3: 16 What does she need to make on Project 4 so that the average for the four projects is 17? Explain your reasoning.

Solution:

One possible solution is to calculate the total number of points needed (17 x 4 or 68) to have an average of 17. She has earned 49 points on the first 3 projects, which means she needs to earn 19 points on Project 4 (68 – 49 = 19).

Example 2

What is the IQR of the data below?



Solution:

The first quartile is 132.5; the third quartile is 142.5. The IQR is 10 (142.5 – 132.5). This value indicates that the values of the middle 50% of the data vary by 10.

Example 3:

The following data set represents the size of 9 families: 3, 2, 4, 2, 9, 8, 2, 11, 4. What is the MAD for this data set?

Solution:

The mean is 5. The MAD is the average variability of the data set. To find the MAD: 1. Find the deviation from the mean. 2. Find the absolute deviation for each of the values from step 1. 3. Find the average of these absolute deviations.

The table below shows these calculations:

Data Value	Deviation from Mean	Absolute Deviation
3	-2	2
2	-3	3
4	-1	1
2	-3	3
9	4	4
8	3	3
2	-3	3
11	6	6
4	-1	1
	MAD	26/9 = 2.89

This value indicates that an average family size varies 2.89 from the mean of 5**.**

Common Misconceptions / Challenges

Students have difficulty choosing appropriate intervals in a histogram. Remind them that intervals must be equal.

Students may mistakenly include outliers in the whiskers on a box and whisker plot. Tell students that outliers do not accurately describe the spread od the data, so they are not included.

Criteria for Success (Performance Level Descriptors)

6.SP.4

Limited: Display simple numerical data using number lines and dot plots
Basic: Display numerical data using number lines and dot plots
Proficient: Display numerical data using number lines, dot plots, histograms, and box plots
Accelerated: Describe and summarize numerical distributions (data sets) by identifying clusters, peaks, gaps, and symmetry, in relationship to the context in which the data were collected; Display and interpret numerical data using number lines, dot plots, histograms, and box plots
Advanced: Describe numerical distributions (data sets) by identifying clusters, peaks, gaps, and symmetry, in relationship to the context in which the data were collected

6.SP.5

Limited:
Basic: Find the median of an even number of whole number data points; find the mean of whole number data points
Proficient: Calculate median, mean, and range
Accelerated: Calculate interquartile range
Advanced: N/A

6.SP.5a

Limited: N/A
Basic: N/A
Proficient: N/A
Accelerated: N/A
Advanced: N/A

6.SP.5b

Limited: N/A
Basic: N/A
Proficient: N/A
Accelerated: N/A
Advanced: N/A

6.SP.5c

Limited: N/A
Basic: N/A
Proficient: N/A
Accelerated: Find mean absolute deviation
Advanced: Use mean absolute deviation to interpret data

6.SP.5d

Limited: N/A
Basic: N/A
Proficient: N/A
Accelerated: Choose the correct measure of center relating to a certain context
Advanced: Choose, calculate, and justify the correct measure of center relating to a certain context

Prior Knowledge	Future Learning
5.MD.2. Display and interpret data in graphs (picture graphs, bar graphs, and line plots) to solve problems using numbers and operations for this grade	7.SP.3.a. Summarize quantitative data sets in relation to their context by using mean absolute deviation (MAD), interpreting mean as a balance point.

Career Connections

http://www.xpmath.com/ca reers/math_topics.php

https://www.statslife.org.uk/